Do the Right Thing?
Mixed Motives and the Condorcet Jury Theorem

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Abstract

We analyze voting in a Condorcet Jury model where voters have both instrumental and expressive motives. Regardless of the relative weight placed on expressive motives, they significantly affect equilibrium voting behavior even in modest-size elections. In large elections, information aggregation crucially and discontinuously depends on the correlation between instrumental and expressive motives. Above some correlation threshold, information fully aggregates, while below this threshold, an election does no better than a coin-flip in determining the correct outcome. Even when information fully aggregates in the limit and the population is ideologically unbiased, increasing the size of the electorate always reduces accuracy over some region.

Keywords: Condorcet, Information Aggregation, Expressive Voting

1 Introduction

Why do so many people bother to vote in large elections? And how effectively do elections aggregate dispersed information? These are among the most important questions in voting theory. Initiated by Downs (1957), the first question has generated a lively literature on the "paradox of voting." This is the idea that if people vote purely to affect the outcome of an election, there is little reason to vote at all, since an individual vote does nothing except in the unlikely case of a tie. This paradox presents a problem for many so-called rational choice voting models. Indeed, many of these models predict extremely low turnout while, in reality, turn out is, of course, quite considerable. (See Feddersen, 2004, for an excellent overview of this literature.)

An obvious solution to the paradox lies in the observation that voters are not exclusively driven by the probability of being pivotal. Citizens also vote because they perceive it to be their civic duty, and because it allows them to give expression to their personal norms, convictions and sentiments. (See, e.g., Riker and Ordeshook, 1968.) That is, voters are not driven by *instrumental motives* alone. At least to some extent, they are also motivated to show up (or not), and vote in a particular way, by their perception of what it means to "do the right thing"—as a citizen, as a partisan, or as a representative. In the literature, the latter considerations are referred to as *expressive motives* and the payoffs associated with them are independent of the outcome of the election.[1]

A literature dating back to Condorcet (1785) has examined the second question—is voting a good mechanism for aggregating information? In that literature, voters are assumed to be only driven by instrumental payoffs—payoffs that are contingent purely on the outcome of the election. Expression does not usually figure into the calculus of voting in these models. Here, the main finding is that voting performs surprisingly well. For a broad class of models, large elections fully aggregate information. (See, e.g., Feddersen and Pesendorfer, 1997.)

An obvious question is how expressive motives affect the information aggregation properties of elections. In this paper, we endeavor to answer this question by studying the Condorcet Jury Theorem when voters have both instrumental as well expressive preferences. We model expressive payoffs as deriving from some norm, which may or may not be influenced by information about the candidates or policy options under consideration. In addition to receiving an instrumental payoff if the objectively better candidate or policy is selected, a voter receives an expressive payoff by voting in line with this norm. We allow the weight on expressive preferences to be arbitrarily small and investigate how voting behavior and information aggregation change with the size of the electorate.

To fix ideas, consider the situation of a union voting on whether to strike. Individual union members have information as to the likelihood that management will back down and the strike will be successful. Each also has norms concerning support of the union. Norms may be formed by solidarity with fathers and grandfathers who also worked for the union. Norms may be influenced by social factors: how can I look my co-workers in the eye if I vote a certain way? Norms may be formed by ideology, by a sense of justice about labor-management power relations, or a host of other factors. When norms are in line with facts—e.g., I think the strike will succeed and my norms say to vote for a strike—the voting calculus is simple. Tension arises, however, when facts and norms collide. A union member may see little hope that the strike will succeed but feel that the governing norm is to vote for a strike. In that case, voting will depend crucially on the size of the voting body: In a large voting body, an indi-

[1] In the case of a private citizen voting in an election, expressive payoffs are mostly intrinsic in nature. That is, they derive from how a vote for A instead of B makes the voter feel about himself. In the case of an elected representative, on the other hand, expressive payoffs are largely extrinsic in nature: They derive from what his constituents think about him as a result of his voting behavior, thereby affecting his chances of reelection.

vidual vote is unlikely to matter and a conflicted member can "free ride" by voting according to the norm rather than the facts. This type of behavior is considerably riskier in a smaller voting body where a vote is more likely to be decisive. How does this tension resolve itself in terms of voting, welfare, and information aggregation?

In the polar case where voters care only about instrumental payoffs, our model is one that would be instantly recognizable to Condorcet. Majority rule performs spectacularly well and information fully aggregates. Outside this polar case, however, we show that the conclusions are much more nuanced. First, a voter with stable preferences and the same information will vote very differently depending on the size of the election. In small elections, such a voter is well-described by a purely instrumental model. He correctly updates his information, conditions on being decisive, and votes accordingly. In large elections, however, even if the voter puts almost no weight on expressive motives, the effect of instrumental motives becomes negligible since the voter correctly perceives that there is little chance that he will cast the tie-breaking vote. Instead, such a voter will rationally vote in purely expressive fashion—doing what he perceives to be the norm, even if it conflicts with his information about instrumental payoffs which constitute, say, 99.9% of his overall utility.

The upshot of this difference in voting behavior in small versus large elections is that, regardless of the weight placed on expressive preferences, voting only produces the "right" outcome when expressive norms and information are sufficiently correlated. Put differently, voting works when voter norms are sufficiently influenced by facts about the issue under consideration. When norms are primarily driven by ideologies that are largely immune to the facts of the issue at hand, we show that voting produces dismal outcomes: In large elections, voting is no better than simply flipping a coin to determine the winner.

Perhaps more surprising is how welfare changes with the size of the electorate. In many purely instrumental models, the decision as to how large the electorate should be is simple: make it as large as possible, since welfare is increasing with size. When voters also have expressive preferences, this is no longer the case. Now, the planner must consider the following trade-off: Increasing the size of the electorate means that, in principle, more information is available to better determine the outcome of the election. However, increasing the electorate also increases the "expressiveness" of equilibrium voting, thus reducing the informativeness of each vote. We show that, even if information fully aggregates in the limit, there always exists a region in which adding more voters reduces welfare. When there is a finite upper bound on the size of the electorate, *disenfranchising* part of the electorate may lead to better outcomes, even when all voters are equally well-informed, and are also otherwise identical ex ante.

To summarize, the main finding of the paper is this: models in which voters have purely instrumental preferences are not "close" to those where voters place some weight on expressive considerations. The voting behavior, information aggregation, and welfare comparative statics can all look very different when expressive preferences figure into the calculus of voting—even if the relative weight of expressive preferences in an individual voter's utility is very small.

The remainder of the paper proceeds as follows: In the remainder of this section, we summarize some of the relevant literature. Section 2 presents the model. In section 3, we describe equilibrium voting behavior in pure strategies. Section 4 adds mixed strategies and fully characterizes all voting equilibria. In section 5, we study the informational and welfare properties of equilibria. Finally, section 6 concludes. Proofs for most results are relegated to an appendix.

Related Literature

The notion that voters are motivated by considerations other than the purely instrumental dates back to Downs (1957). Riker and Ordeshook (1968) offer an early formalization by adding their famous D (duty) term to the voting calculus. Versions of this idea have appeared in many analyses explaining voter turnout, of which Feddersen and Sandroni (2002) offers perhaps the most compelling recent example.[2] More broadly, mixed motives in voting have been investigated in a variety of settings. See, e.g., Razin (2003) and Callander (2008). Coate, Conlin, and Moro (2008) and Coate and Conlin (2004) present convincing empirical evidence for the importance of non-instrumental considerations in voting.

Our focus on expressive preferences builds closely on Brennan and Lomasky (1993). In their seminal book, they present an intuitive analysis of the effect of expressive preferences on voting behavior and outcomes. In our view, their work has not received the consideration it deserves in the field of economics, perhaps because of the lack of a formal mathematical model. An important contribution of our paper is to fill in this gap by providing a formal modeling framework for voting when voters have mixed motives.[3]

Our paper also contributes to the vast literature on information aggregation in voting. The polar case of our model where expressive considerations are completely absent is a special case of Feddersen and Pesendorfer (1998). That paper, as well as Feddersen and Pesendorfer (1997), shows that a version of the Condorcet Jury Theorem holds quite generally—large elections succeed in aggregating information.[4] However, all of these results are distinguished by the fact that preferences are purely instrumental.

[2] Their model is inspired by the work of Harsanyi (1977, 1980, 1992) on "rule-utilitarianism."

[3] See also Harsanyi (1969), Tullock (1971), Brennan and Buchanan (1984). Kliemt (1986), and Kirchgaessner and Pommerehne (1993).

[4] For similar results, see also Mclennan (1998) and Myerson (1998). Bhattacharya (2008), on the other hand, offers a negative result. He analyzes a class of instrumental models in which information does not aggregate.

2 Model

We study a simple model of elections, where information aggregation is a primary consideration and where voters have both instrumental and expressive preferences. Suppose that there are two equally likely states, labeled $\theta \in \{\alpha, \beta\}$, and an election with two possible outcomes, $o \in \{A, B\}$. Each of the $n+1$ voters, where n is even, receives a conditionally independent signal, $s \in \{a, b\}$. With probability $r \in \left(\frac{1}{2}, 1\right)$, a voter receives a "true" signal (i.e. receives an a signal when the state is α and a b signal when the state is β). Otherwise, the voter receives a "false" signal, defined in analogous fashion.

Voters' payoffs are determined by the outcome of the election, the state, and their individual votes. Outcome A is better for all voters in state α, while outcome B is better in state β. Specifically, all voters receive a payoff equal to 1 if the correct outcome is selected and a payoff equal to 0 if the incorrect outcome is selected. We shall refer to this aspect of voter preferences as their *instrumental payoffs.* Voters also care about "doing the right thing" according to some norm. This norm may be intrinsic—i.e., a personal view of what is right—or it may be extrinsic—e.g., a representative may have to "explain his vote" to constituents back home. Regardless, doing the right thing consists of casting a vote consistent with that norm and yields a payoff equal to 1. Casting a vote against one's norm yields a payoff equal to zero. We shall refer to this aspect of voter preferences as their *expressive payoffs.* Finally, let ε denote the relative weight a voter places on expressive payoffs, while complementary weight is placed on instrumental payoffs.

Next, we turn to how norms are determined. Suppose that, ex ante, norms are such that, with probability $\rho \geq \frac{1}{2}$, a given voter views voting for A as normative.[5] After the state has been realized and the voter receives his signal, his view about the appropriate norm might change. Specifically, we suppose that with probability $q \in [0, 1)$ a voter is influenced by his new information and adopts a norm consistent with his (posterior) beliefs about which outcome is more likely to be superior. Thus, a voter receiving an a signal adopts voting for A as the norm with probability q, while, with the same probability, a voter with a b signal adopts voting for B as the norm. With the complementary probability $1-q$, however, the voter sticks to his ex ante norm. One can think of q as representing the propensity of norms to be influenced by the facts. A voter's norm is summarized by his *type* t, $t \in \{A, B\}$. An A type voter perceives voting for A as doing the right thing, while a B type voter perceives voting for B as doing the right thing. With probability q a voter's type is determined by his signal (i.e., a signal of a induces type A and likewise for b). With probability $1-q$ a voter is not influenced by his signal, such that his type and signal are uncorrelated. In that case, the voter's type is A with probability ρ.

To summarize, a voter with type t who casts a vote v, $v \in \{A, B\}$, in an election

[5] Assuming $\rho \geq \frac{1}{2}$ is without loss of generality. For the opposite case, simply relabel the outcomes.

that produces outcome o in state θ receives payoffs

$$U = \begin{cases} 1 & \text{if} \quad o \text{ is right and } v = t \\ (1-\varepsilon) & \text{if} \quad o \text{ is right and } v \neq t \\ \varepsilon & \text{if} \quad o \text{ is wrong and } v = t \\ 0 & \text{if} \quad o \text{ is wrong and } v \neq t \end{cases}$$

To fix ideas, consider a local election for a primarily technical role, such as City Controller. All voters benefit by having the more competent candidate take the post, though they might differ in their views as to who is more competent. Voters are also influenced by norms which, in this case, often center around party affiliation. Many voters find identity in supporting Republican candidates and policies over a broad swath of issues. Suppose that such a voter receives a signal that the Republican candidate for Controller has the better credentials, more experience, and so on. Clearly, his view about the right candidate to vote for will be unchanged. If, on the other hand, he learns that the Republican candidate is deficient in the skills required to fill the post, his attitude toward what constitutes doing the right thing might change—he might, for the purposes of the Controller race, take the practical view that the right thing is to vote for the better candidate rather than simply voting Republican. Of course, other Republican voters may be unwilling or unable to put aside their party identity, regardless of the facts of the particular case. The model tries to capture the idea that, for some voters, norms are malleable depending the facts of the case, while for others, they are not. In the end, a voter's payoffs are determined by both instrumental factors—the competence of the elected candidate—and expressive factors—whether his vote was consistent with his norms. The parameter ε captures the weighting of expressive relative to instrumental factors.

To determine the outcome of the election, all $n+1$ voters simultaneously cast their votes. The outcome is decided by majority rule. When determining equilibrium voting behavior, we restrict attention to symmetric responsive strategies that are undominated. An equilibrium is then characterized by the voting behavior of each kind of voter, i.e., voters with signals $s \in \{a, b\}$ and types $t \in \{A, B\}$. Absent expressive preferences, which corresponds to $\varepsilon = 0$, this model is quite standard and easy to analyze. If voters have purely instrumental preferences, in equilibrium, they vote according to their signals and, for large n, the probability that the correct outcome is selected converges to one.[6]

We may divide voters into four classes depending on the realizations of s and t. When s and t coincide, i.e. $s = a$ and $t = A$, or $s = b$ and $t = B$, we say that a voter is *unconflicted.* When s and t differ, we say that a voter is *conflicted.* After some simplification, it may be readily shown that the probability of being conflicted is equal to $\frac{1}{2}(1-q)$. The parameter q is a measure of the correlation between a voter's signal and his type. Notice that, when $q = 1$, type and signal are perfectly correlated

[6]Because both outcomes are equally likely to be correct ex ante, the usual worries about strategic voting highlighted by Austen-Smith and Banks (1996) are absent in this special case

and, as a consequence, there are no conflicted voters. As q falls, the probability that a voter is conflicted increases and reaches a maximum of 50% at $q = 0$. Thus, conflicted voters are always a minority of the voting population.

We now turn to voting strategy. Let γ_α denote the equilibrium probability that, in state α, a randomly chosen voter casts a vote for A. Likewise, let γ_β denote the probability of an A vote in state β. We first show that voting for unconflicted voters is straightforward—they simply cast a vote consistent with both their signal and their type. Formally,

Lemma 1 *In all symmetric responsive equilibria, unconflicted voters vote according to their type and signal.*

The voting behavior of conflicted voters is considerably more complex (and interesting) to describe. Before proceeding with an equilibrium characterization, it is useful to define strategies more formally. Let σ_s denote the probability that a conflicted voter with signal s votes for A. From Lemma 1 it follows that

$$\gamma_\alpha = qr + r(1-q)\rho + r(1-q)(1-\rho)\sigma_a + (1-r)(1-q)\rho\sigma_b \quad (1)$$

$$\gamma_\beta = q(1-r) + (1-r)(1-q)\rho + (1-r)(1-q)(1-\rho)\sigma_a + r(1-q)\rho\sigma_b \quad (2)$$

Note that $\gamma_\beta < \gamma_\alpha$ for all σ_a and σ_b. That is, A receives a greater (expected) share of the vote when it is the superior option than when it is the inferior option. The same is true for B. While σ_s describes a generic mixed strategy, two polar cases are of interest. When $\sigma_a = 1$ and $\sigma_b = 0$, we say that a voter votes *instrumentally*—i.e. purely according to his signal. Similarly, when $\sigma_a = 0$ and $\sigma_b = 1$, we say that a voter votes *expressively*—i.e. purely according to his type. The difference in expected payoffs for a conflicted voter with signal s who votes instrumentally as opposed to expressively takes on the same sign as V_s, where

$$V_a \equiv \binom{n}{\frac{n}{2}} \left(r\left(\gamma_\alpha(1-\gamma_\alpha)\right)^{\frac{n}{2}} - (1-r)\left(\gamma_\beta(1-\gamma_\beta)\right)^{\frac{n}{2}} \right) - \frac{\varepsilon}{1-\varepsilon}$$

and

$$V_b \equiv \binom{n}{\frac{n}{2}} \left(r\left(\gamma_\beta(1-\gamma_\beta)\right)^{\frac{n}{2}} - (1-r)\left(\gamma_\alpha(1-\gamma_\alpha)\right)^{\frac{n}{2}} \right) - \frac{\varepsilon}{1-\varepsilon}$$

Intuitively, instrumental payoff differences arise only when the election is tied. They reflect the balance between tilting the election toward the correct outcome given the signal, versus tilting the election toward the incorrect outcome. Expressive payoff differences, on the other hand, always arise. Here, the term $\frac{\varepsilon}{1-\varepsilon}$ represents the (normalized) cost of voting against one's type.

3 Equilibrium Voting in Pure Strategies

Having characterized the equilibrium voting behavior of unconflicted voters, we now turn to the behavior of conflicted voters. As we show below, the equilibrium voting behavior of conflicted voters will typically vary with the size of the electorate. Intuitively, as the size of the electorate grows, instrumental considerations, which hinge on the probability of being pivotal, become less important and voting becomes more expressive.

While $n+1$ denotes the (discrete) size of the electorate, it is sometimes convenient to use a continuous analog of n, which we denote by m. We also adapt the usual floor/ceiling notation for the integer parts of m to reflect the restriction that n be an even number. Specifically, let $\lfloor m \rfloor$ be the largest even integer less than or equal to m, and let $\lceil m \rceil$ be the smallest even integer greater than or equal to m. We use the Gamma function to extend factorials to non-integer values. Recall that, for integer values, $n! = \Gamma(n+1)$ and, hence, $\binom{n}{\frac{n}{2}} = \frac{\Gamma(n+1)}{\Gamma^2\left(\frac{n}{2}+1\right)}$. The expression $\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}$ represents the continuous analog. This makes the function V_s and other expressions well-defined for all values of m. For instance,

$$V_b = \frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)} \left\{ r\left(z_\beta\right)^{\frac{m}{2}} - (1-r)\left(z_\alpha\right)^{\frac{m}{2}} \right\} - \frac{\varepsilon}{1-\varepsilon}$$

where $z_\theta = \gamma_\theta\left(1-\gamma_\theta\right)$.

We now offer a very useful technical lemma which shows that, for fixed values of γ_α and γ_β, V_s is monotone in m. Formally,

Lemma 2 *Fix z_α and z_β such that $0 < z_\alpha \leq z_\beta \leq \frac{1}{4}$. Then*

$$\Phi(m) \equiv \frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)} \left\{ r\left(z_\beta\right)^{\frac{m}{2}} - (1-r)\left(z_\alpha\right)^{\frac{m}{2}} \right\}$$

is strictly decreasing in m.

Moreover, $\lim_{m\to\infty} \Phi(m) \downarrow 0$.

Instrumental Equilibrium

From an information aggregation perspective, it would be ideal if voters simply voted in line with their signals. As we have shown above, this is not a problem for unconflicted voters. For conflicted voters, whether to vote instrumentally turns on whether the gains from voting instrumentally and improving the probability of breaking a tie in the correct direction outweigh the losses from voting against one's expressive preferences. From Lemma 2 we know that the benefits from instrumental voting are strictly decreasing in m. Thus, the largest size electorate for which instrumental voting is an equilibrium amounts to determining the value of m such that $V_a|_{\sigma_a=1,\sigma_b=0} = V_b|_{\sigma_a=1,\sigma_b=0} = 0$, or, equivalently,

$$\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)} (2r-1)\left(r(1-r)\right)^{\frac{m}{2}} = \frac{\varepsilon}{1-\varepsilon} \quad (3)$$

Lemma 2 implies that for all $m > 0$,

$$\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}(2r-1)(r(1-r))^{\frac{m}{2}} < 2r-1$$

Hence, a necessary condition for instrumental voting to be an equilibrium for some size of the electorate is that $\frac{\varepsilon}{1-\varepsilon} < 2r-1$ or, equivalently, $\varepsilon < \frac{1}{r}\left(r-\frac{1}{2}\right)$. If $\varepsilon \geq \frac{1}{r}\left(r-\frac{1}{2}\right)$, voting expressively is the unique equilibrium regardless of the size of the electorate. The remainder of the analysis excludes this rather uninteresting case. Formally,

Assumption 1: $\varepsilon < \frac{1}{r}\left(r-\frac{1}{2}\right)$.

Assumption 1 together with Lemma 2 guarantees that equation (3) has a *unique* solution, which we shall denote by $\bar{m}_I > 0$. The expression $\bar{m}_I$ represents the largest size voting body where conflicted voters vote purely according to their signals. Formally,

Proposition 1 *Purely instrumental voting is an equilibrium iff* $n \leq \bar{m}_I$.

The Proposition says that, for large voting bodies, instrumental voting is not an equilibrium. Since the probability of being pivotal declines as the number of voters increases, the *effective* weight of instrumental payoffs, which depends on the chance of a tied election, declines relative to expressive payoffs. Once voters are sufficiently unlikely to swing the election, they are better of voting according to their type, locking in the ε expressive utility, rather than voting according to their signal and foregoing this sure gain for a lottery with only a small chance of success.

Inspection of equation (3) reveals that $\bar{m}_I$ does not depend on q and ρ. That is, the electoral size for which instrumental voting is an equilibrium is independent of the degree of correlation between instrumental and expressive motives and the level of ex ante bias in expressive motives. It is also interesting to note that the maximal size of the electorate for which instrumental voting is an equilibrium varies non-monotonically with the quality of voters' information: When voters are poorly informed, i.e. $r < \frac{1}{2(1-\varepsilon)}$, instrumental voting is never an equilibrium. However, as voters become perfectly informed—i.e. $r \to 1$—$\bar{m}_I$ also goes to zero. There are two different forces at work here. When r is low, a voter is likely to be pivotal but unlikely to push the outcome in the right direction with his vote; hence expected instrumental payoffs are low. When r is high, a voter is very likely to push the outcome in the correct direction but very unlikely to be pivotal. Again, this leads to low expected instrumental payoffs. Thus, the electorate sizes for which instrumental voting is an equilibrium are largest when voters are moderately well informed.

The fact that instrumental voting is not an equilibrium for elections larger than $\bar{m}_I$ might seem of no consequence if the weight on instrumental payoffs is small (i.e. $\varepsilon \to 0$). Indeed, inspection of equation (3) reveals that $\bar{m}_I$ becomes infinite as ε goes to zero. The key question is the *rate* at which $\bar{m}_I$ grows as ε shrinks. While $\bar{m}_I$ does not have a closed-form solution, a good approximation is available. This approximation allows us to examine the relationship between $\bar{m}_I$ and ε.

Remark 1 *For small ε,*

$$\bar{m}_I \approx \frac{W\left(\frac{-\ln(4r(1-r))}{\frac{\pi}{2}\left(\frac{1}{r}\frac{\varepsilon}{1-\varepsilon}\right)^2}\right)}{-\ln\left(4r\left(1-r\right)\right)} \tag{4}$$

where $W(\cdot)$ is the Lambert W function.[7]

Consider the sequence $\varepsilon_k = \frac{1}{k}$. Substituting this expression into equation (4) yields the sequence $\bar{m}_{I,k} \approx \xi W\left((k-1)^2\right)$, where ξ is a scaling factor independent of k. Now recall that $\lim_{k\to\infty} \frac{\ln k}{W(k)} = 1$. Hence, we can conclude that $\bar{m}_{I,k}$ grows only at rate $2\ln k$ as ε_k falls. In other words, while $\bar{m}_{I,k}$ increases, it does so only extremely slowly. For instance, if $r = \frac{3}{5}$ and 95% weight is placed on instrumental payoffs, then instrumental voting is an equilibrium for electorates numbering no more than 7 people. If, instead, we assume that 99.9% weight is placed on instrumental payoffs, then the maximum electorate size for which instrumental voting is an equilibrium increases only to 129 people.

Expressive equilibrium Let us now turn to the polar opposite case—purely expressive voting. Expressive voting is an equilibrium if and only if $\sigma_a = 0$ and $\sigma_b = 1$ is optimal for conflicted voters. Let γ_α^E be equal to γ_α—as defined in equation (1)—with $\sigma_a = 0$ and $\sigma_b = 1$. Let γ_β^E be likewise defined. It may be readily verified that $\gamma_\alpha^E\left(1-\gamma_\alpha^E\right) < \gamma_\beta^E\left(1-\gamma_\beta^E\right)$. For expressive voting to comprise an equilibrium requires that $V_a \leq 0$ and $V_b \leq 0$. Notice that

$$\begin{aligned} V_b|_{\sigma_a=0,\sigma_b=1} &= \frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left\{r\left(\gamma_\beta^E\left(1-\gamma_\beta^E\right)\right)^{\frac{m}{2}} - (1-r)\left(\gamma_\alpha^E\left(1-\gamma_\alpha^E\right)\right)^{\frac{m}{2}}\right\} - \frac{\varepsilon}{1-\varepsilon} \\ &> \frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left\{r\left(\gamma_\alpha^E\left(1-\gamma_\alpha^E\right)\right)^{\frac{m}{2}} - (1-r)\left(\gamma_\beta^E\left(1-\gamma_\beta^E\right)\right)^{\frac{m}{2}}\right\} - \frac{\varepsilon}{1-\varepsilon} \\ &= V_a|_{\sigma_a=0,\sigma_b=1} \end{aligned}$$

Thus, we need only check the incentive condition for expressive voting for voters with b signals. Because $\gamma_\alpha^E\left(1-\gamma_\alpha^E\right) < \gamma_\beta^E\left(1-\gamma_\beta^E\right)$, Lemma 2 implies that the benefits from expressive voting are increasing in m. Hence, finding the smallest size electorate for which expressive voting is an equilibrium amounts to determining the value of m such that $V_b|_{\sigma_a=0,\sigma_b=1} = 0$, or, equivalently,

$$\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left\{r\left(\gamma_\beta^E\left(1-\gamma_\beta^E\right)\right)^{\frac{m}{2}} - (1-r)\left(\gamma_\alpha^E\left(1-\gamma_\alpha^E\right)\right)^{\frac{m}{2}}\right\} = \frac{\varepsilon}{1-\varepsilon} \tag{5}$$

[7] Recall that the Lambert W function is the inverse of $f(W) = W\exp(W)$.

Assumption 1 together with Lemma 2 guarantees that equation (5) has a *unique* solution, which we shall denote by $\underline{m}_E > 0$.[8] The value $\underline{m}_E$ represents the smallest size voting body for which conflicted voters find it optimal to vote purely expressively. Formally,

Proposition 2 *Expressive voting is an equilibrium iff* $n \geq \underline{m}_E$.

One might have thought that $\underline{m}_E = \bar{m}_I$, i.e., once instrumental voting ceases to be an equilibrium, expressive voting becomes an equilibrium. Notice, however, that this is (generically) not the case. This is most easily seen for $q = 0$. In that case, equation (5) reduces to

$$\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}(2r-1)(\rho(1-\rho))^{\frac{m}{2}} = \frac{\varepsilon}{1-\varepsilon}$$

Lemma 2 implies that $\underline{m}_E < \bar{m}_I$ for $\rho > r$, whereas $\underline{m}_E > \bar{m}_I$ for $\rho < r$. Indeed, the gap between $\bar{m}_I$ and $\underline{m}_E$ can be quite large. To see this, let us return to the example above, adding the remaining parameters of the model.

Example 1 *Suppose that* $r = 3/5$, $\rho = 3/4$, $q = 3/4$, *and* $\varepsilon = \frac{1}{1000}$. *Then instrumental voting is an equilibrium for 129 voters or less, while expressive voting is an equilibrium for 5,863 voters or more.*

This leaves open the question of what happens in between instrumental and expressive voting. The next section fills in this gap by considering mixed strategies, in order to characterize all equilibria for every n.

4 Full Equilibrium Characterization

We now add the possibility of mixed strategies. The following lemma allows us to narrow down the types of conflicted voter behavior that can arise in equilibrium.

Proposition 3 *The following and only the following kinds of equilibria can arise:*

1. *Instrumental: In this class, all voters vote purely according to their signals, i.e.,* $\sigma_a = 1$ *and* $\sigma_b = 0$.

2. *Expressive: In this class, all voters vote purely according to their types, i.e.,* $\sigma_a = 0$ *and* $\sigma_b = 1$.

[8] While $\underline{m}_E$ does not admit a closed-form solution, a good approximation is available: $\underline{m}_E \approx \dfrac{W\left(\dfrac{-\ln\left(4\gamma_\beta^E\left(1-\gamma_\beta^E\right)\right)}{\frac{\pi}{2}\left(\frac{1}{r}\frac{\varepsilon}{1-\varepsilon}\right)^2}\right)}{-\ln\left(4\gamma_\beta^E\left(1-\gamma_\beta^E\right)\right)}$

3. *Completely mixed: In this class, unconflicted voters vote according to their signals, while conflicted voters randomize with strictly positive probability between voting according to their signals and voting according to their types. I.e.,* $\sigma_a, \sigma_b \in (0,1)$.

4. *Partially-mixed: In this class, unconflicted voters vote according to their signals (or types), conflicted voters with a signals vote according to their types, while conflicted voters with b signals mix. I.e.,* $\sigma_a = 0, \sigma_b \in (0,1)$.

To characterize all equilibria, it is convenient to divide the exercise into two parts: equilibrium when the correlation between expressive preferences and signals is high, and when that correlation is low. We shall say that correlation is high when $q > q^*$, where q^* is defined below. Correlation is low when $q < q^*$. We show that for high correlation, there always exists a unique equilibrium for each n. For low correlation, there may be multiple equilibria for some n. Multiplicity can occur both within an equilibrium class as well as across equilibrium classes. For example, two different partially-mixed equilibria may coexist for the same n, while at the same time there also exists an expressive equilibrium.

Uniqueness turns on the monotonicity of V_b. Essentially, if q is such that V_b is increasing in σ_b at $\sigma_b = 1$ and $m = \underline{m}_E$, then equilibrium is unique for every n. Formally, q^* is defined as the (unique) value

$$q^* \equiv \max \left\{ q \in [0,1] \left. \frac{dV_b}{d\sigma_b} \right|_{\sigma_a=0,\sigma_b=1,m=\underline{m}_E(q)} = 0 \right\}$$

where $\underline{m}_E(q)$ reflects the dependence of $\underline{m}_E$ on the degree of correlation. With this definition in mind, we have:

Lemma 3 *q^* exists and is unique.*
Furthermore, $q_0 < q^* < q_1$, *where* $q_0 = \frac{1}{2}\frac{2\rho-1}{\rho-r(1-r)}$ *and* $q_1 = \frac{1}{2}\frac{2\rho-1}{\rho-(1-r)}$.

The bounds on q^* are useful in two ways. First, while q^* does not admit a closed form solution, the bounds are easily calculated. Second, and more importantly, q_1 is intimately connected with information aggregation, as we will show in Section 5.

4.1 High Correlation

This section's main result is to show that, when correlation is high, there is a unique equilibrium for every n. Moreover, as n increases, the equilibrium sequence moves smoothly from instrumental to expressive voting. When voting bodies are small, purely instrumental voting is the unique equilibrium. As voting bodies grow larger, equilibrium voting becomes completely mixed. As they grow larger yet, voters with a signals vote expressively while voters with b signals continue to mix; however the latter are increasingly likely to vote expressively. Finally, in sufficiently large electorates, purely expressive voting is the unique equilibrium.

Completely-mixed equilibrium Let us first consider completely-mixed equilibria. The following lemma identifies properties that all such equilibria share:

Lemma 4 *In any completely-mixed equilibrium,*

1. *The probability of being pivotal is the same in states α and β, independent of n, and equal to $\frac{1}{2r-1}\frac{\varepsilon}{1-\varepsilon}$.*
2. *The probability that a vote conforms to the underlying state is the same in both states and strictly greater than $\frac{1}{2}$. Formally, $\gamma_\alpha = 1 - \gamma_\beta > \frac{1}{2}$.*
3. $\sigma_a = 1 - \frac{\rho}{1-\rho}\sigma_b$

Since $\sigma_a = 1 - \frac{\rho}{1-\rho}\sigma_b$, notice that the largest value σ_b can take under a completely-mixed equilibrium is $\sigma_b = \frac{1-\rho}{\rho}$, in which case $\sigma_a = 0$. Since $q > q^*$, V_b is increasing in σ_b. Hence, among completely-mixed equilibria, V_b takes on its highest value at the boundary point $(\sigma_a, \sigma_b) = \left(0, \frac{1-\rho}{\rho}\right)$. Moreover, from Lemma 2 we know that the benefit from instrumental voting is decreasing in m. Finally, under a completely-mixed equilibrium, a voter must be indifferent between voting instrumentally and voting expressively. Combining these observations yields an upper bound on the size of the electorate for which a completely-mixed equilibrium exists, which we denote by $\bar{m}_{CM}$. Formally, $\bar{m}_{CM}$ is the (unique) solution to

$$V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} = 0$$

The argument for existence and uniqueness of $\bar{m}_{CM}$ is similar to that for $\underline{m}_E$.[9] Since V_b is increasing in σ_b, V_b takes on its lowest value for $\sigma_b = 0$, in which case $\sigma_a = 1$. This constellation of σs corresponds to instrumental voting. It follows from Lemma 2 that voters are *not* indifferent between instrumental and expressive voting when $m < \bar{m}_I$. Hence,

Proposition 4 *Under high correlation, a completely-mixed equilibrium exists iff n is such that $\bar{m}_I < n < \bar{m}_{CM}$. For each n, there exists exactly one mixed-strategy equilibrium. Moreover, $\bar{m}_{CM} > \bar{m}_I$.*

Since there exists a unique completely-mixed equilibrium for every n in the interval $\bar{m}_I < n < \bar{m}_{CM}$, we can define a sequence of completely-mixed equilibria, with n running from $\lceil \bar{m}_I \rceil < n < \lfloor \bar{m}_{CM} \rfloor$. Note that this sequence is fully characterized by the sequence of mixing probabilities $\{\{\sigma_a, \sigma_b\}_n\}_{\lceil \bar{m}_I \rceil < n < \lfloor \bar{m}_{CM} \rfloor}$. We say that voting becomes more expressive if σ_a decreases and σ_b increases. We now show that the completely-mixed equilibrium becomes more expressive as n increases.

Proposition 5 *In the completely-mixed equilibrium sequence, voting becomes more expressive as n increases.*

---

[9] While $\bar{m}_{CM}$ does not admit a closed-form solution, a good approximation is available: $\bar{m}_{CM} \approx \frac{W\left(\frac{-\ln\left(4\gamma_\alpha^{CM}\left(1-\gamma_\alpha^{CM}\right)\right)}{\frac{\pi}{2}\left(\frac{1}{2r-1}\frac{\varepsilon}{1-\varepsilon}\right)^2}\right)}{-\ln(4\gamma_\alpha^{CM}(1-\gamma_\alpha^{CM}))}$, where $\gamma_\alpha^{CM} = \gamma_\alpha|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}}$.

Partially-mixed equilibrium We now turn to partially-mixed equilibria. In a partially-mixed equilibrium, conflicted voters with a signals vote expressively while those with b signals mix. Since $q > q^*$, we know that V_b is increasing in σ_b. V_b is smallest when $(\sigma_a, \sigma_b) = \left(0, \frac{1-\rho}{\rho}\right)$ and largest when $(\sigma_a, \sigma_b) = (0, 1)$. By arguments analogous to those establishing the bounds on completely-mixed equilibria, this fact together with Lemma 2 implies:

Proposition 6 *Under high correlation, a partially-mixed equilibrium exists iff n is such that $\bar{m}_{CM} \leq n < \underline{m}_E$. For each n, there exists exactly one partially-mixed equilibrium. Moreover, $\bar{m}_{CM} < \underline{m}_E$.*

Thus, we have shown that, as the size of the electorate continues to increase, we move from a completely-mixed equilibrium to one in which only conflicted voters with b signals mix, while conflicted voters with a signals vote expressively. Since there is a unique partially-mixed equilibrium for every n in the interval $\bar{m}_{CM} \leq n < \underline{m}_E$, we can define a sequence of partially-mixed equilibria, with n running from $\lceil \bar{m}_{CM} \rceil < n < \lfloor \underline{m}_E \rfloor$. Note that this sequence is fully characterized by the sequence of mixing probabilities $\{\{\sigma_b\}_n\}_{\lceil \bar{m}_{CM} \rceil < n < \lfloor \underline{m}_E \rfloor}$. We now show that the partially-mixed equilibrium becomes more expressive as n increases. Formally,

Proposition 7 *In the partially-mixed equilibrium sequence, as n increases, voting becomes more expressive.*

Summary Note that, under high correlation, the intervals for which the various classes of equilibria exist partition the set of even integers. Moreover, as equilibrium within each class is unique for every n, we have shown that

Proposition 8 *Under high correlation, there exists a unique equilibrium for each n. As n increases, we move from:*

1. *Instrumental voting for $n \leq \bar{m}_I$; to*
2. *Completely-mixed voting for $\bar{m}_I < n < \bar{m}_{CM}$; to*
3. *Partially-mixed voting for $\bar{m}_{CM} \leq n < \underline{m}_E$; to*
4. *Expressive voting for $n \geq \underline{m}_E$*

Proposition 8 establishes that, for small elections, instrumental voting is the unique equilibrium. At larger sizes, both types of conflicted voters mix between instrumental and expressive voting. Eventually, conflicted voters with a signals vote expressively while conflicted voters with b signals continue to mix. In the end, equilibrium consists of a pure strategy profile in which everyone votes expressively. As equilibrium is unique for each n, Proposition 8 allows us to define an infinite equilibrium sequence indexed by n, which we denote by C_0.

We have seen before that, within each equilibrium class, voting becomes (weakly) more expressive as n increases. Moreover, it is easily verified that voting also becomes more expressive when we move from one equilibrium class in C_0 to the next. Hence, we have shown that

Proposition 9 *Under high correlation, when n increases, equilibrium behavior becomes more expressive.*

Finally, let us return to Example 1. Because $q = \frac{3}{4} > q_1 > q^*$, we are in the high correlation case and the analysis above applies. Recall that, for the parameter values in the example, instrumental voting is an equilibrium for 129 voters or less, while expressive voting is an equilibrium for 5,863 voters or more. Completely-mixed voting is an equilibrium for electorate sizes between 131 and 163, while partially-mixed voting is an equilibrium for electorate sizes between 165 and 5,861.

4.2 Low Correlation

We now turn to the case where correlation between types and signals is low, i.e. $q < q^*$. As we will see, this makes equilibrium behavior more varied. While the classes of equilibria are the same as under high correlation, under low correlation, the ranges for which these classes exist may overlap. Indeed, an instrumental and an expressive equilibrium may coexist for the same value of n. Moreover, equilibrium may no longer be unique within a class: For generic parameter values, two different partially-mixed equilibria coexist. These partially-mixed equilibria form two separate sequences, which we call the "low" partially-mixed equilibrium sequence and the "high" partially-mixed equilibrium sequence. Formally,

Definition 1 *A low partially-mixed equilibrium sequence is a sequence of partially-mixed equilibria running from $\lceil \bar{m}_{CM} \rceil$ to $\lfloor \bar{m}_{PM} \rfloor$ (defined below), where, if we ignore the integer constraint, σ_b starts at $\sigma_b = \frac{1-\rho}{\rho}$, increases in n, and ends at $\sigma_b = \sigma_{b,\bar{m}_{PM}}$ (defined below).*

Definition 2 *A high partially-mixed equilibrium sequence is a sequence of partially-mixed equilibria running from $\lceil \underline{m}_E \rceil$ to $\lfloor \bar{m}_{PM} \rfloor$, where, if we ignore the integer constraint, σ_b starts at $\sigma_b = 1$, decreases in n, and ends at $\sigma_b = \sigma_{b,\bar{m}_{PM}}$.*

The end point $\bar{m}_{PM}$ of high and low partially-mixed voting is implicitly defined as the largest value of m such that the indifference condition for conflicted voters with a b signals still has a solution in σ_b, i.e.,

$$\bar{m}_{PM} \equiv \max \left\{ m \middle| \, V_b|_{\sigma_a = 0} = 0 \text{ has a solution in } \sigma_b \in \left[\frac{1-\rho}{\rho}, 1 \right] \right\}$$

Here, $\sigma_{b,\bar{m}_{PM}}$ is defined as that solution.[10]

Recall that when $q > q^*$, V_b was increasing in σ_b. This guaranteed two things: (1) there was a unique partially-mixed equilibrium and (2) partially-mixed equilibria ended when expressive equilibria began. Neither of these properties is true anymore under low correlation. Indeed, $V_b|_{\sigma_a=0}$ is single peaked in σ_b over the interval $\left[\frac{1-\rho}{\rho}, 1\right]$. Since a partially-mixed equilibrium occurs at a value of σ_b where $V_b = 0$, single-peakedness of $V_b|_{\sigma_a=0}$ implies that there will typically be two partially-mixed equilibria under low correlation. Moreover, both sequences of partially-mixed equilibria end when the peak of $V_b|_{\sigma_a=0}$ takes on a value of zero. In that case, expressive voting will already be an equilibrium, since away from the peak, at $\sigma_b = 1$ (i.e. under expressive voting), V_b will already be negative. This is the intuition for the following lemma, which provides sufficient conditions for expressive voting to overlap with partially-mixed voting.

Lemma 5 *$\bar{m}_{PM}$ exists and is unique. Moreover, for $q \leq q_0$, $\underline{m}_E < \bar{m}_{PM}$.*

The low partially-mixed equilibrium sequence under low correlation corresponds to the unique partially-mixed equilibrium sequence under high correlation, differing only in its upper endpoint. Lemma 5 offers conditions that guarantee that this sequence overlaps, in part, with expressive voting. The low partially-mixed equilibrium sequence still has the property that the degree of expressiveness is increasing in the size of the electorate. In contrast, the high partially-mixed equilibrium sequence is entirely new. It can only arise under low correlation. Moreover, it has the somewhat counter-intuitive property that expressiveness *decreases* in the size of the electorate. In a sense, this sequence is the mirror image of the low partially-mixed equilibrium sequence: The two sequences converge to one another and they coincide at the boundary point $\bar{m}_{PM}$.

We are now in a position to fully characterize equilibria under low correlation.

Proposition 10 *Under low correlation,the following and only the following equilibria arise:*

1. *Purely instrumental voting is an equilibrium iff $n \leq \overline{m}_I$.*
2. *Completely-mixed voting is an equilibrium iff $\overline{m}_I < n < \bar{m}_{CM}$.*
3. *Low partially-mixed voting is an equilibrium iff $\bar{m}_{CM} \leq n < \bar{m}_{PM}$.*

   Moreover, for each n, there exists exactly one such equilibrium.
4. *High partially-mixed voting is an equilibrium iff $\underline{m}_E \leq n < \bar{m}_{PM}$.*

   Moreover, for each n, there exists exactly one such equilibrium.

[10] While neither $\bar{m}_{PM}$ nor $\sigma_{b,\bar{m}_{PM}}$ admit closed-form solutions, approximations are available: For small ε, $\bar{m}_{PM} \approx \frac{2}{\pi}\left(r\frac{1-\varepsilon}{\varepsilon}\right)^2$and $\sigma_{b,\bar{m}_{PM}} \approx \frac{\frac{1}{2}-q(1-r)}{r(1-q)\rho} - \frac{1-r}{r}$.

5. *Expressive voting is an equilibrium iff* $n \geq \underline{m}_E$.

In Proposition 10, together, the equilibria *1.*, *2.* and *3.* constitute a finite equilibrium sequence consisting of a concatenation of instrumental equilibria for small n, completely-mixed equilibria for somewhat larger n and, finally, low partially-mixed equilibria. We denote this sequence by C_1. We denote by C_2 the set of two, partially overlapping, sequences of equilibria *4.* and *5.*, namely, the infinite sequence of expressive equilibria and the finite sequence of high partially-mixed equilibria. Both sequences start at $\underline{m}_E$, but the latter only runs up to $\bar{m}_{PM}$, while the former runs all the way to ∞. Notice that, when $\bar{m}_{PM}$ coincides with $\underline{m}_E$ (as is always the case when correlation is high), together, C_1 and C_2 partition the space of electorate sizes. When $\bar{m}_{PM}$ and $\underline{m}_E$ do not coincide (as is always the case for $q \leq q_0$), then, for $\underline{m}_E \leq n < \bar{m}_{PM}$, equilibria *4.* and *5.* coexist with one of the equilibria *1.*, *2.*, and *3.* In particular, for some parameter values, instrumental and expressive equilibria can coexist. To see this, consider the following amendment of Example 1, where we have reduced q from 3/4 to 1/7.

Example 2 *Suppose that* $r = 3/5$*,* $q = 1/7$*,* $\rho = 3/4$*, and* $\varepsilon = 1/1000$*. Then, instrumental voting is an equilibrium for* $n \leq 128$*, while expressive voting is an equilibrium for* $n \geq 48$*. There is a completely-mixed equilibrium for* $130 \leq n \leq 330$*. There is a low partially-mixed equilibrium for* $332 \leq n \leq 228{,}724$*, while there is a high partially-mixed equilibrium for* $48 \leq n \leq 228{,}724$*.*

Expressive Preferences and the Probability of Being Pivotal

Once the probability of casting a decisive vote falls sufficiently, expressive motives completely crowd out instrumental motives. Hence, one might suspect that, in our model, pivotality considerations play a subordinated role more generally. This, however, is not the case.

Figure 1 illustrates the probability of casting the pivotal vote in Example 2. As the figure shows, while the probability of being pivotal falls rapidly under purely instrumental voting, it is constant under a completely-mixed equilibrium.[11] It then falls slowly under a partially-mixed equilibrium, but remains stubbornly high.[12] As the figure indicates, the probability of casting a decisive vote under partially-mixed voting stays above 0.08%, even when n is as much as 228,724. As a comparison, when $\varepsilon = 0$ (i.e., purely instrumental preferences), the chance of being pivotal at that n is 1.57×10^{2027} times smaller. Beyond $\bar{m}_{PM}$, only expressive voting is an equilibrium and the chance of being pivotal falls discontinuously to essentially zero.

The large difference in pivot probabilities between $\varepsilon > 0$ *and* $\varepsilon = 0$ *does not depend on low correlation. To see this, note that in Example 1 the pivot probability at* n_E

[11]Indeed, recall from Lemma 4 that the probability of being pivotal in the completely-mixed equilibrium is equal to $\frac{1}{2r-1}\frac{\varepsilon}{1-\varepsilon}$.

[12]It can be shown that the probability of being pivotal in the low and high partially-mixed equilibria converges to $\frac{1}{2}\frac{\varepsilon}{1-\varepsilon}\frac{1}{r}$.

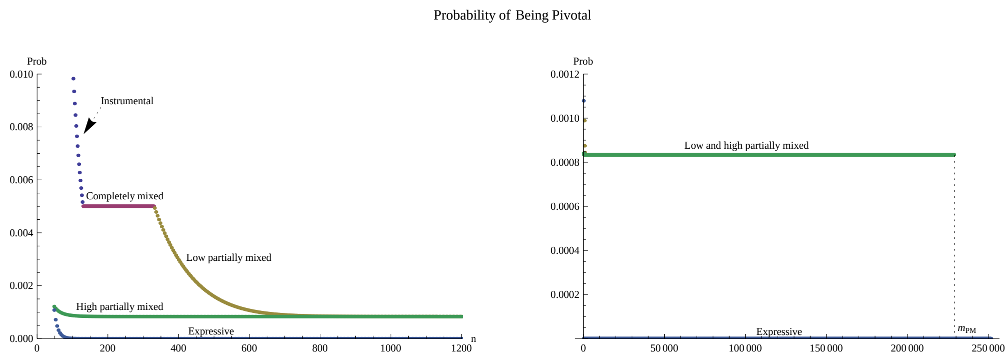

Figure 1:

is again 0.08%, *while at that* n *the pivot probality under* $\varepsilon = 0$ *is* 1.1347×10^{-54}, *i.e.,*1.36×10^{51} *times smaller.*

5 Optimal Size of the Electorate

The analysis above points out a tension that arises when voters also have expressive preferences. On the one hand, increasing the size of the electorate is helpful since, collectively, voters have more information on which to base decisions as to the better outcome. On the other hand, increasing the size of the electorate undercuts a voter's incentive to vote instrumentally, and thereby dilutes the information contained in each vote. Thus, unlike settings where only instrumental motives are present, the addition of expressive preferences introduces the possibility that the optimal size of the electorate may well be bounded—i.e., that beyond a certain point, the additional information from increasing the number of voters gets crowded out by increased degrees of expressive voting. It is useful to define the *selection accuracy*, S, of a voting equilibrium to be the probability that the correct outcome is selected. For a fixed electorate size, the accuracy of all voting equilibria may be unambiguously ordered, as the following lemma shows.

Lemma 6 *If multiple equilibria exist for given* n*, then they can be ranked in terms of their selection accuracy:*

$$S \in \{S_I, S_{CM}, S_{LPM}\} > S_{HPM} > S_E$$

Lemma 6 is intuitive: The ranking corresponds perfectly with the expressiveness of the equilibrium. Thus, a purely expressive equilibrium is least informative, while a purely instrumental equilibrium (provided one exists for the same size electorate) is most informative. Other equilibria are similarly ordered.

Of course, a social planner may not solely be concerned with the accuracy of election outcomes. Since voters derive utility from voting expressively, these payoffs should also be included in any welfare calculation. We first show that there is no conflict between maximizing accuracy and maximizing welfare. Formally, if we use per capita ex ante expected payoff as our measure of welfare, which we denote by W, we have the following equivalence result:

Proposition 11 *Welfare and accuracy rankings of equilibria coincide.*

Specifically,

1. *If multiple equilibria exist for given n, then the welfare ranking of equilibria coincides with the information aggregation ranking. That is, for fixed n,*

$$W \in \{W_I, W_{CM}, W_{LPM}\} > W_{HPM} > W_E$$

2. *Assume that the best equilibrium is selected. Welfare improves iff accuracy improves.*

Proposition 11 says that the welfare and accuracy properties of voting equilibria coincide. Thus, for the remainder of the analysis, we study the optimal size of the electorate in terms of accuracy.

Asymptotics

Information fully aggregates if and only if A enjoys a majority vote share in state α while B enjoys a majority vote share in state β. That is, when $n \to \infty$, it must be that $\gamma_\alpha > \frac{1}{2}$ and $\gamma_\beta < \frac{1}{2}$. Our equilibrium characterization revealed that, as n grows arbitrarily large, there always is a unique equilibrium, namely, expressive voting. Hence, the limiting vote shares exist and are given by:

$$\begin{aligned} \gamma_\alpha &= qr + (1-q)\,\rho \\ \gamma_\beta &= (1-r)\,q + (1-q)\,\rho \end{aligned}$$

Since $r > \frac{1}{2}$ and $\rho \geq \frac{1}{2}$, we have that $\gamma_\alpha > \frac{1}{2}$.[13] Hence, in state α, the correct outcome occurs with probability 1 in the limit. Note, however, that $\gamma_\beta < \frac{1}{2}$ if and only if $q > q_1$. Since $q^* < q_1$, this means that under low correlation, the correct outcome *never* occurs in state β. Moreover, high correlation is, by itself, not sufficient to guarantee information aggregation. Instead correlation must be "very high" (i.e. $q > q_1$) for information to fully aggregate. Thus, we have shown

Proposition 12 *Large elections fully aggregate information if and only if correlation is very high (i.e., $q > q_1$).*

Otherwise, large elections are no better than a coin flip at selecting the correct outcome.

[13] Note that $\gamma_\alpha = \frac{1}{2}$ in the non-generic case where $q = 0$ and $\rho = \frac{1}{2}$, which we ignore.

Proposition 12 suggests that conclusions about the desirable informational properties of large elections hinge critically on two assumptions: either voters place no weight at all on expressive preferences; or correlation between expressive motives and signals is sufficiently high that, even though voting is purely expressive, votes retain enough information to produce correct outcomes in the limit. The degree of correlation required depends on the degree of asymmetry in the ex ante norms (i.e. ρ). When ρ is high, i.e. norms are pre-disposed toward outcome A, a significant degree of correlation is required to overcome this bias. On the other hand, when ρ is close to one-half, i.e. voters are evenly divided in their predispositions toward A and B, even minimal correlation is sufficient to produce the correct result.

We are now in a position to more precisely state what we consider to be a main contribution of the paper: Even when the weight on expressive preferences becomes arbitrarily small, as the electorate grows, equilibrium accuracy and welfare under mixed motives may not approach accuracy and welfare under purely instrumental motives. To see this, fix a sequence $\varepsilon_k \to 0$. For each element of this sequence, let S_{ε_k} denote the asymptotic voting accuracy as $n \to \infty$. For n large, expressive voting is the unique equilibrium. Hence, for every ε_k, $S_{\varepsilon_k} = \lim_{n\to\infty} S_E$. Finally, let S^* denote the limit equilibrium accuracy as $n \to \infty$ for $\varepsilon = 0$ (i.e., when voters have purely instrumental preferences). In our model, it is easy to show that $S^* = 1$. Using Proposition 12, it then follows that

Proposition 13 *Unless correlation is very high, asymptotic accuracy as* $\varepsilon \to 0$ *does not converge to asymptotic accuracy for* $\varepsilon = 0$*. Formally, if* $q < q_1$*, then for every sequence* $\varepsilon_k \to 0$*,* $S_{\varepsilon_k} \to \frac{1}{2} < S^* = 1$*.*

Finite Sized Electorates

When correlation is high, the problem of determining the optimal size of the electorate would seem trivial—simply select the largest possible electorate. Indeed, absent expressive preferences, this is exactly the case in our model. Additional voters have no impact on the informativeness of a given vote and, hence, the probability of getting a correct outcome is always increasing in the size of the electorate.

When expressive motives are present, however, this is no longer the case. While it is still true that, in the instrumental equilibrium region, increasing the size of the electorate is helpful, the following example illustrates that this is no longer true for completely-mixed equilibria. Specifically, suppose that we amend Example 2 to remove any asymmetry in ex ante norms, i.e., $\rho = 1/2$. Since $q > q_1 = 0$, correlation is high. Hence, there is a unique equilibrium for every n and information fully aggregates in the limit. However, as Figure 2 illustrates, increasing the number of voters is not the same as increasing informativeness. While accuracy increases as long as there is an instrumental equilibrium (up to $n = 128$ in the figure), it falls when n is between 130 and 2,730. Beyond this point, welfare once again increases but only reaches its previous high water mark for $n = 6{,}472$. The region of decreasing accuracy corresponds to electorate sizes where there is only a completely-mixed equilibrium.

Here, the information losses from more expressive voting outpace the gains from the increased number of voters. The region from $n = 2{,}732$ onwards corresponds to expressive voting.[14] The informativeness of votes no longer degenerates as n increases and, since $q > q_1$, additional votes improve equilibrium accuracy.

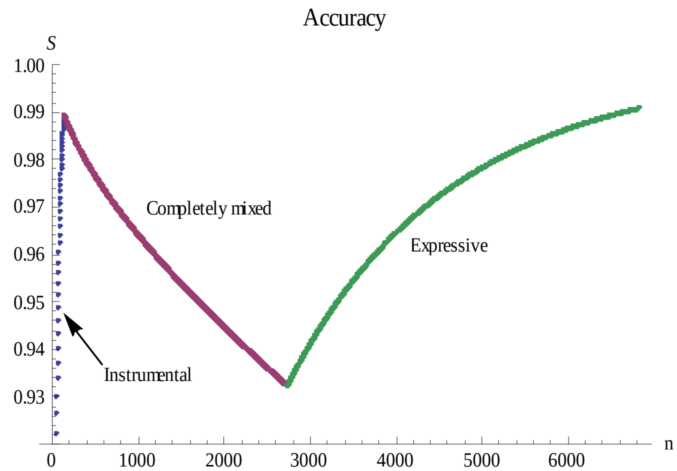

Figure 2:

One may wonder if this pattern depends crucially on the particular parameter values chosen. We conjecture (based on many such examples) that there always is a region of decreasing equilibrium accuracy. Define $S(m)$ to be the equilibrium accuracy of a size $m+1$ electorate.[15] We have:

Conjecture 1 *Informativeness decreases in the region of the completely-mixed equilibrium. Formally,* $S(\overline{m}_I) > S(\overline{m}_{CM})$.

The conjecture implies that it is better to have $\overline{m}_I + 1$ voters voting instrumentally than $\overline{m}_{CM} + 1$ voters voting expressively, regardless of whether information aggregates in the limit.

We now study accuracy properties under low correlation. We first show that, for a sufficiently large electorate, accuracy is always decreasing in the size of the electorate. To derive this result, first notice that for $n > \overline{m}_{PM}$, expressive voting is the unique equilibrium. Next, the following lemma shows that, for every n and every equilibrium, the right outcome is selected more than half the time. Formally,

Lemma 7 *Fix an equilibrium* γ_α, γ_β. *Then, for every* n, $S(n) > \frac{1}{2}$.

[14] When $\rho = \frac{1}{2}$, the partially-mixed equilibrium region disappears as a consequence of the full symmetry of the model.

[15] Since m is not integer valued, we use the continuous analog of informativeness, which corresponds to a regularized Beta function.

Now recall that, under low correlation, the accuracy of the expressive equilibrium sequence converges to one-half. It then follows from Lemma 7, that accuracy converges to one-half from above. Hence, we have shown:

Proposition 14 *Under low correlation, for n sufficiently large, equilibrium accuracy is strictly decreasing in the size of the electorate.*

Perhaps more striking is the fact that equilibrium accuracy drops discontinuously in m. In other words, accuracy does not degrade "gracefully" as the electorate grows, but falls off a cliff. Formally, fix the sequence of most informative equilibria $C^*(m)$ as a function of the electorate size m, which we treat as continuous. From Proposition 6 we know that this sequence is uniquely defined even in the presence of multiple equilibria. Next, notice that, in the neighborhood of $m = \overline{m}_{PM}$, the accuracy of this sequence jumps discontinuously downward from S_{LPM} to S_E. Thus we have shown,

Proposition 15 *Under low correlation, equilibrium accuracy falls discontinuously at $\overline{m}_{PM}$.*

Proposition 15 points out the possibility of a sudden collapse in equilibrium accuracy as the electorate expands—even if voters manage to always coordinate on the most informative equilibrium. The accuracy properties of equilibria under low correlation are nicely captured in the following example. Specifically, Figure 3 depicts equilibrium accuracy of the equilibria in Example 2 for different ranges of n. As the figure reveals, accuracy is increasing in n under instrumental voting, decreases in the region of the completely-mixed equilibrium, is hump-shaped under the low partially-mixed equilibrium sequence, and falls discontinuously from .75 to .5 at 228,725 voters. Since correlation is low, adding more voters beyond 228,725 simply produces worse results.

As Figure 3 highlights, once partially-mixed equilibria cease to exist, accuracy falls from about 75% to about 50%. This is, in fact, a general property. Since A types are voting expressively, for large n, there is only a negligible chance that a voter is pivotal in state α. This implies that the second term of V_b, namely $(1-r)\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{m}{2}}$, is essentially zero. Hence, V_b is maximized at $\gamma_\beta = \frac{1}{2}$, which must then be the value of γ_β at the threshold size $\bar{m}_{PM}$ where partially-mixed equilibria cease to exist. Thus, at this threshold, outcome A is occurring with probability near one in state α, while the outcome is a near coin-toss in state β. Therefore, accuracy is very close to 75%. Once the electorate grows beyond this threshold, only expressive voting remains a possibility and outcome A is selected in *both* states with probability close to one. Hence, accuracy falls to essentially 50%.

6 Conclusion

Since Condorcet, perhaps the main message from the "informational" voting literature is the remarkable ability of elections to aggregate information and produce the

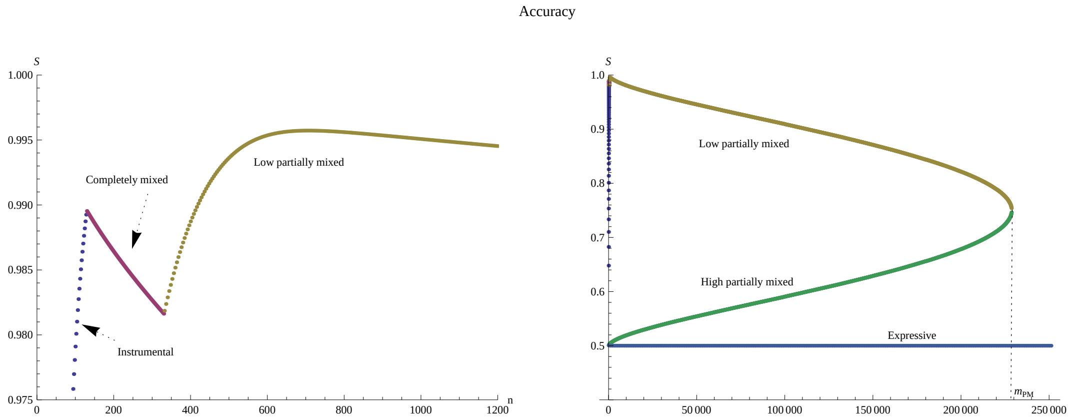

Figure 3:

correct outcome. Our analysis suggests that we have, perhaps, been overly optimistic in these conclusions. When we enrich the classical model by admitting the possibility that voters might be motivated by expressive concerns as well as instrumental motives, the results are more ambiguous. As long as expressive preferences are sufficiently malleable—facts influence voter norms—then the original conclusions still hold. However, if the correlation between expressive preferences and information falls below a critical threshold, we reach a more pessimistic conclusion: Large elections can, and will, produce dismal outcomes. What is most worrying is the fact that this conclusion holds even if voters place only negligible weight on expressive preferences.

While the existing voting literature by and large concludes that increasing the size of the electorate is unambiguously positive for the quality of election outcomes, our analysis suggests that constitution designers need to consider a more subtle trade-off. While expanding the electorate increases the amount of information in the system, it also increases the incentives for voters to free ride and engage in expressive voting. As we have shown, the latter effect can outweigh the former and lead to a situation where increasing the size of the electorate leads to worse outcomes. Thus, our analysis casts doubt on the optimistic view about the ability of majority rule to produce good societal outcomes in large elections.

At a broader level, our results uncover a trade-off that constitution designers need to consider. Our analysis suggests that, in a representative democracy, limiting the size of the electorate selecting a representative can greatly improve outcomes. However, this merely moves the problem up one level. Increasing the number of representatives creates its own problems for policy making, as representatives will be tempted to vote more expressively in larger legislatures. This trade-off does not arise in purely instrumental models.

The potential problems associated with mixed motives may be seen in US electoral institutions. Since the founding of the country, the populations of US counties and cities have grown dramatically. As a consequence, so have the size of electorates choosing county and city officials. The same is true at the state and national levels. For example, the number of citizens per congressional district has risen from an average of 30,000 in 1790 to almost 700,000 as of 2008. The developments in the UK are even more dramatic. After the Reform Act of 1832, an electorate of around 720,000 voters chose 658 members of Parliament, for an average of approximately 1100 voters per MP. In 2010, the electorate has grown to around 45,000,000 people, while the number of MPs has fallen to 650 MPs, resulting in an average of 69,000 voters per MP. These sharp rises in electorate sizes have increased the incentives for free riding by voters. More so than in the past, voters can indulge their expressive preferences without worry that the vote they cast might be decisive.

A Proofs

Proof of Proposition 1:

Consider an unconflicted voter with signal a. The difference in his payoffs from voting for A as compared to voting for B

$$\binom{n}{\frac{n}{2}}(1-\varepsilon)\left(r\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}-(1-r)\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}\right)+\varepsilon$$

Here, the first term denotes the difference in probabilities of pushing the election in the "right" direction versus pushing it in the wrong direction, multiplied times $(1-\varepsilon)$, which is the payoff of doing so. The second term, ε, denoted the assured payoff of voting according to one's type.

Suppose, contrary to the statement of the lemma, that this expression is negative. That is,

$$\binom{n}{\frac{n}{2}}(1-\varepsilon)\left(r\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}-(1-r)\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}\right)+\varepsilon\leq 0 \tag{6}$$

First, note that a necessary condition for this inequality to hold is that $\gamma_\beta\left(1-\gamma_\beta\right)>\gamma_\alpha\left(1-\gamma_\alpha\right)$.

Second, note that the inequality implies that a conflicted voter with an a signal would also strictly prefer to vote for B, since the difference in that voters payoffs from voting for A versus voting for B is given by

$$\begin{aligned}
&\binom{n}{\frac{n}{2}}(1-\varepsilon)\left(r\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}-(1-r)\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}\right)-\varepsilon\\
<\ &\binom{n}{\frac{n}{2}}(1-\varepsilon)\left(r\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}-(1-r)\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}\right)+\varepsilon\\
\leq\ &0
\end{aligned}$$

Furthermore, an unconflicted voter with a b signal would strictly prefer to vote for B, since the difference in that voters payoffs from voting for B compared to voting for A is given by

$$\begin{aligned}
& \binom{n}{\frac{n}{2}}(1-\varepsilon)\left(r\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}-(1-r)\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}\right)+\varepsilon \\
> & \binom{n}{\frac{n}{2}}(1-\varepsilon)\left((1-r)\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}-r\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}\right)+\varepsilon \\
> & \binom{n}{\frac{n}{2}}(1-\varepsilon)\left((1-r)\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}-r\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}\right)-\varepsilon \\
\geq & \ 0
\end{aligned}$$

Here, the first inequality uses $r > \frac{1}{2}$, while the last inequality uses the fact that, by assumption (and as expressed by equation 6), an unconflicted voter with signal a strictly prefers to vote for B.

Finally, a conflicted voter with a b signal would strictly prefer to voter for B since

$$\begin{aligned}
& \binom{n}{\frac{n}{2}}(1-\varepsilon)\left(r\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}-(1-r)\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}\right)-\varepsilon \\
> & \binom{n}{\frac{n}{2}}(1-\varepsilon)\left((1-r)\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}-r\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}\right)-\varepsilon \\
> & \ 0
\end{aligned}$$

Hence, we have shown that if an unconflicted voter with signal a weakly prefers to vote for B, then all voters would strictly prefer to vote for candidate B. In turn, this implies that $\gamma_\alpha = \gamma_\beta = 0$. This, however, contradicts $\gamma_\beta\left(1-\gamma_\beta\right) > \gamma_\alpha\left(1-\gamma_\alpha\right)$. We may then conclude that an unconflicted voter with signal a strictly prefers to vote for A.

The proof that an unconflicted voter with signal b strictly prefers to vote for B is entirely analogous.

Proof of Lemma 2:

Differentiating $\Phi(m)$ with respect to m, we obtain

$$\Phi'(m) = \frac{1}{2}\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left(\begin{array}{c} (1-r)(z_\alpha)^{\frac{m}{2}}\left(2H\left[\frac{m}{2}\right]-2H[m]-\log[z_\alpha]\right) \\ -r(z_\beta)^{\frac{m}{2}}\left(2H\left[\frac{m}{2}\right]-2H[m]-\log[z_\beta]\right)\end{array}\right)$$

where $H[x]$ is the xth harmonic number. This expression takes the sign of

$$(1-r)(z_\alpha)^{\frac{m}{2}}\left(2H\left[\frac{m}{2}\right]-2H[m]-\log[z_\alpha]\right)-r(z_\beta)^{\frac{m}{2}}\left(2H\left[\frac{m}{2}\right]-2H[m]-\log[z_\beta]\right)$$

which is negative iff

$$\frac{1-r}{r}\left(2\left(H\left[\frac{m}{2}\right]-H[m]\right)-\log[z_\alpha]\right) < \left(\frac{z_\beta}{z_\alpha}\right)^{\frac{m}{2}}\left(2\left(H\left[\frac{m}{2}\right]-H[m]\right)-\log[z_\beta]\right)$$

We claim that $2\left(H\left[\frac{m}{2}\right]-H\left[m\right]\right)-\log\left[z_\beta\right]>0$, for all $m\geq 2$. To see this, note that for $m=2$,

$$\begin{aligned} 2\left(H\left[1\right]-H\left[2\right]\right)-\log\left[z_\beta\right] &> 2\left(H\left[1\right]-H\left[2\right]\right)-\log\left[\frac{1}{4}\right] \\ &= 2\left(1-\frac{3}{2}\right)-\log\left[\frac{1}{4}\right] \\ &> 0 \end{aligned}$$

Because $H\left[m\right]$ is concave in m, the inequality then also holds for all $m>2$.

This implies that $\Phi'\left(m\right)<0$ iff

$$\frac{1-r}{r}\frac{2\left(H\left[m\right]-H\left[\frac{m}{2}\right]\right)+\log\left[z_\alpha\right]}{2\left(H\left[m\right]-H\left[\frac{m}{2}\right]\right)+\log\left[z_\beta\right]}<\left(\frac{z_\beta}{z_\alpha}\right)^{\frac{m}{2}}$$

And this inequality indeed holds, because $r<\frac{1}{2}$, and $z_\alpha\leq z_\beta$.

The second part of the lemma, i.e., $\lim_{m\to\infty}\Phi\left(m\right)=0$, follows from Stirling's approximation of $\Gamma\left[m+1\right]$ for large m:

$$\begin{aligned} & \frac{\Gamma\left(m+1\right)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left\{r\left(z_\beta\right)^{\frac{m}{2}}-\left(1-r\right)\left(z_\alpha\right)^{\frac{m}{2}}\right\} \\ \approx\; & \frac{\sqrt{2\pi m}\frac{m^m}{e^m}}{\left(\sqrt{2\pi\frac{m}{2}}\frac{\frac{m}{2}^{\frac{m}{2}}}{e^{\frac{m}{2}}}\right)^2}\left\{r\left(z_\beta\right)^{\frac{m}{2}}-\left(1-r\right)\left(z_\alpha\right)^{\frac{m}{2}}\right\} \\ =\; & \frac{\sqrt{2\pi m}\frac{m^m}{e^m}}{\pi m\frac{\frac{m}{2}^m}{e^m}}\left\{r\left(z_\beta\right)^{\frac{m}{2}}-\left(1-r\right)\left(z_\alpha\right)^{\frac{m}{2}}\right\} \\ =\; & \sqrt{2}\frac{2^m}{\sqrt{\pi m}}\left\{r\left(z_\beta\right)^{\frac{m}{2}}-\left(1-r\right)\left(z_\alpha\right)^{\frac{m}{2}}\right\} \\ =\; & \sqrt{2}\left(r\frac{\left(2\sqrt{z_\beta}\right)^m}{\sqrt{\pi m}}-\left(1-r\right)\frac{\left(2\sqrt{z_\alpha}\right)^m}{\sqrt{\pi m}}\right) \end{aligned}$$

Now note that both terms converge to zero as $m\to\infty$, because $z_\alpha\leq z_\beta\leq\frac{1}{4}$.

Proof of Proposition 1

The necessary and sufficient condition for instrumental voting to be an equilibrium is that

$$\left(1-\varepsilon\right)\left(2r-1\right)\binom{n}{\frac{n}{2}}\left(r\left(1-r\right)\right)^{\frac{n}{2}}\geq\varepsilon$$

We may rearrange this inequality as

$$\binom{n}{\frac{n}{2}}\left(2r-1\right)\left(r\left(1-r\right)\right)^{\frac{n}{2}}\geq\frac{\varepsilon}{1-\varepsilon}$$

Now, note that lemma 2 with $z_\alpha = z_\beta = r(1-r)$ implies that the LHS is strictly decreasing in n. As a consequence, $\binom{n}{\frac{n}{2}}(2r-1)(r(1-r))^{\frac{n}{2}} \geq \frac{\varepsilon}{1-\varepsilon}$ holds iff $n \leq \bar{m}_I$, where $\bar{m}_I$ is the value of m that solves the (continuous analogue of the) expression with equality.

Proof of Remark 1:

$\bar{m}_I$ solves

$$\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)}(2r-1)(r(1-r))^{\frac{m}{2}} = \frac{\varepsilon}{1-\varepsilon} \tag{7}$$

For ε small, clearly, m has to be large. For large m, using Sterling's approximation, we have

$$\frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)} \approx \frac{\sqrt{2\pi m}\frac{m^m}{e^m}}{\left(\sqrt{2\pi\frac{m}{2}}\frac{\frac{m}{2}^{\frac{m}{2}}}{e^{\frac{m}{2}}}\right)^2} = \frac{\sqrt{2}\frac{m^m}{e^m}}{\sqrt{\pi m}\frac{\frac{m}{2}^m}{e^m}}$$

Hence, approximately, Equation (7) becomes

$$\frac{\left(2\sqrt{r(1-r)}\right)^m}{\sqrt{m}} = \sqrt{\frac{\pi}{2}}\frac{1}{2r-1}\frac{\varepsilon}{1-\varepsilon}$$

Now, solving for m gives

$$\bar{m}_I \approx \frac{1}{\ln\left(\frac{1}{4r(1-r)}\right)}\text{LambertW}\left(\frac{\ln\left(\frac{1}{4r(1-r)}\right)}{\frac{\pi}{2}\left(\frac{1}{2r-1}\frac{\varepsilon}{1-\varepsilon}\right)^2}\right)$$

This completes the proof.

Lemma 8 *If $\sigma_a = 0$ and $\sigma_b > \frac{1-\rho}{\rho}$ then $\gamma_\alpha(1-\gamma_\alpha) < \gamma_\beta(1-\gamma_\beta)$*

Proof. It is sufficient to show that $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$.

If $\sigma_a = 0$ and $\sigma_b > \frac{1-\rho}{\rho}$, then

$$\begin{aligned}
\gamma_\alpha &> qr + r(1-q)\rho + (1-r)(1-q)(1-\rho) \\
&= qr + (1-q)(2r\rho - \rho - r + 1) \\
&\geq qr + (1-q)\frac{1}{2} \\
&> \frac{1}{2}
\end{aligned}$$

where the first inequality follows from $\sigma_b > \frac{1-\rho}{\rho}$, the second from $\rho \geq \frac{1}{2}$ and the third from $r > \frac{1}{2}$.

Now, if $\gamma_\beta > \frac{1}{2}$, then $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$ follows immediately from the fact that $\gamma_\beta < \gamma_\alpha$.

If $\gamma_\beta \leq \frac{1}{2}$, then $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$ is equivalent to showing that $\gamma_\alpha - \left(1-\gamma_\beta\right) > 0$. For $\sigma_a = 0$ and $\sigma_b > \frac{1-\rho}{\rho}$,

$$\begin{aligned}\gamma_\alpha - \left(1-\gamma_\beta\right) &= qr + r\left(1-q\right)\rho + \left(1-q\right)\rho\sigma_b + q\left(1-r\right) + \left(1-r\right)\left(1-q\right)\rho - 1 \\ &> qr + r\left(1-q\right)\rho + \left(1-q\right)\left(1-\rho\right) + q\left(1-r\right) + \left(1-r\right)\left(1-q\right)\rho - 1 \\ &= 0\end{aligned}$$

This completes the proof. ■

Proof of Proposition 2:

Under expressive voting, we claim that $V_a < V_b$. Thus, we need only check the incentive condition to vote expressively for voters with b signals. To see this, notice that

$$\begin{aligned}V_b|_{\sigma_a=0,\sigma_b=1} &= \binom{n}{\frac{n}{2}}\left\{r\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}} - \left(1-r\right)\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}\right\} - \frac{\varepsilon}{1-\varepsilon} \\ &> \binom{n}{\frac{n}{2}}\left\{r\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}} - \left(1-r\right)\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}\right\} - \frac{\varepsilon}{1-\varepsilon} \\ &= V_a|_{\sigma_a=0,\sigma_b=1}\end{aligned}$$

where we have used the fact that, when $\sigma_a = 0$ and $\sigma_b = 1$, it may be readily verified that $\gamma_\alpha\left(1-\gamma_\alpha\right) < \gamma_\beta\left(1-\gamma_\beta\right)$.

The incentive constraint to vote expressively for conflicted voters with b signals is

$$\binom{n}{\frac{n}{2}}\left\{r\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}} - \left(1-r\right)\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}\right\} \leq \frac{\varepsilon}{1-\varepsilon}$$

By construction, at $\underline{m}_E$ the incentive constraint holds with equality. By Lemma 2, the left-hand side of this expression is strictly decreasing in n. Hence, for all $n \geq \underline{m}_E$ the incentive constraint also holds. This completes the proof.

Proof of Remark ??:

$\underline{m}_E$ solves

$$\frac{\Gamma\left(m+1\right)}{\Gamma^2\left(\frac{m}{2}+1\right)}\left(r\left(\tilde{\gamma}_\beta\left(1-\gamma_\beta^A\right)\right)^{\frac{n}{2}} - \left(1-r\right)\left(\tilde{\gamma}_\alpha\left(1-\tilde{\gamma}_\alpha\right)\right)^{\frac{n}{2}}\right) - \frac{\varepsilon}{1-\varepsilon} = 0$$

First note that for ε small, m has to be large. Next, by Lemma 8, $\tilde{\gamma}_\beta\left(1-\tilde{\gamma}_\beta\right) > \tilde{\gamma}_\alpha\left(1-\tilde{\gamma}_\alpha\right)$, such that for large m, $\left(\frac{\tilde{\gamma}_\alpha\left(1-\tilde{\gamma}_\alpha\right)}{\tilde{\gamma}_\beta\left(1-\gamma_\beta^A\right)}\right)^{\frac{m}{2}} \to 0$. Hence, $\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{m}{2}}$ is negligible relative to $\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{m}{2}}$. Therefore,

$$V_b|_{\sigma_a=0,\sigma_b=1} \approx \frac{\Gamma\left(m+1\right)}{\Gamma^2\left(\frac{m}{2}+1\right)}r\left(\tilde{\gamma}_\beta\left(1-\tilde{\gamma}_\beta\right)\right)^{\frac{m}{2}} - \frac{\varepsilon}{1-\varepsilon}$$

Finally, using Sterling's approximation and solving for m gives the result.

Proof of Proposition 3:

The fact that each of these kinds of equilibria can indeed arise is proved by example. (See, e.g., Example 2.) The proof that no other kinds of equilibria can arise proceeds as follows. First, from Lemma 1, we know that all unconflicted voters vote according to their signals. This implies that all equilibria are fully characterized by the mixing probabilities $(\sigma_a, \sigma_b) \in [0,1]^2$ of conflicted voters. To prove the proposition, we have to show that there neither exist equilibria with $\{\sigma_a = 1,\ \sigma_b \in (0,1)\}$, nor with $\{\sigma_a \in (0,1),\ \sigma_b = 1\}$, nor with $\{\sigma_a \in (0,1),\ \sigma_b = 0\}$. This is proved in Lemmas 9, 10, and 11, respectively. which can be found below.

Lemma 9 *There is no partially-mixed equilibrium where $\sigma_a = 1$ and $\sigma_b \in (0,1)$.*

Proof. Suppose, by contradiction, that such an equilibrium does exist.

We first show that $\sigma_a = 1$ implies that $\gamma_\alpha (1-\gamma_\alpha) < \gamma_\beta \left(1-\gamma_\beta\right)$. To establish this, it is sufficient to show that $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$.

One may readily verify that for $\sigma_a = 1$, $\gamma_\alpha > \frac{1}{2}$. Furthermore, it may be readily shown that $\gamma_\beta > \frac{1}{2}$ iff

$$\sigma_b > \frac{r - \frac{1}{2}}{r(1-q)\rho}$$

When $\sigma_b > \frac{r-\frac{1}{2}}{r(1-q)\rho}$ (and, hence, $\gamma_\alpha > \frac{1}{2}$ and $\gamma_\beta > \frac{1}{2}$), $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$ follows immediately from the easily verified fact that $\gamma_\beta < \gamma_\alpha$.

When $\sigma_b \leq \frac{r-\frac{1}{2}}{r(1-q)\rho}$ (and, hence, $\gamma_\alpha > \frac{1}{2}$ and $\gamma_\beta \leq \frac{1}{2}$), $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$ is equivalent to showing that $\gamma_\alpha - \left(1-\gamma_\beta\right) > 0:$

$$\begin{aligned}
\gamma_\alpha - \left(1-\gamma_\beta\right) &= qr + r(1-q) + (1-r)(1-q)\rho\sigma_b - 1 \\
&\quad + ((1-r)q + (1-r)(1-q) + r(1-q)\rho\sigma_b) \\
&= 1 + (1-q)\rho\sigma_b - 1 \\
&= (1-q)\rho\sigma_b \\
&> 0
\end{aligned}$$

Next, we note that $\gamma_\alpha (1-\gamma_\alpha) < \gamma_\beta \left(1-\gamma_\beta\right)$ implies $V_b > V_a$. Observing that $V_a \geq 0$ because $\sigma_a = 0$, we may then conclude that $V_b > V_a \geq 0$. Finally, we note that $V_b > 0$ is incompatible with $\sigma_b \in (0,1)$. This concludes the proof. ■

Lemma 10 *There is no partially-mixed equilibrium where $\sigma_a \in (0,1)$, and $\sigma_b = 1$.*

Proof. Suppose, by contradiction, that such an equilibrium does exist.

We first show that $\sigma_b = 1$ implies $\gamma_\alpha (1-\gamma_\alpha) < \gamma_\beta \left(1-\gamma_\beta\right)$. To establish this, it is sufficient to show that $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$.

One may readily verify that, for $\sigma_b = 1$, $\gamma_\alpha > \frac{1}{2}$. Furthermore, it may be readily shown that $\gamma_\beta > \frac{1}{2}$ iff

$$\sigma_a > \frac{\frac{1}{2} - (1-r)\,q - (1-q)\,\rho}{(1-r)\,(1-q)\,(1-\rho)}$$

When $\sigma_a > \frac{\frac{1}{2}-(1-r)q-(1-q)\rho}{(1-r)(1-q)(1-\rho)}$ (and, hence, $\gamma_\alpha > \frac{1}{2}$ and $\gamma_\beta > \frac{1}{2}$), $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$ follows immediately from the fact that $\gamma_\beta < \gamma_\alpha$.

When $\sigma_a \leq \frac{\frac{1}{2}-(1-r)q-(1-q)\rho}{(1-r)(1-q)(1-\rho)}$(and, hence, $\gamma_\alpha > \frac{1}{2}$ and $\gamma_\beta < \frac{1}{2}$), $\left|\gamma_\alpha - \frac{1}{2}\right| > \left|\gamma_\beta - \frac{1}{2}\right|$ is equivalent to showing that $\gamma_\alpha - \left(1-\gamma_\beta\right) > 0:$

$$\begin{aligned}
\gamma_\alpha - \left(1-\gamma_\beta\right) &= qr + (1-q)\,\rho + r\,(1-q)\,(1-\rho)\,\sigma_a - 1 \\
&\quad + ((1-r)\,q + (1-q)\,\rho + (1-r)\,(1-q)\,(1-\rho)\,\sigma_a) \\
&= q + 2\,(1-q)\,\rho + 2r\,(1-q)\,(1-\rho)\,\sigma_a - 1 \\
&= (1-q)\,(2\rho + 2r\,(1-\rho)\,\sigma_a - 1) \\
&> 0
\end{aligned}$$

Next, we note that $\gamma_\alpha\,(1-\gamma_\alpha) < \gamma_\beta\left(1-\gamma_\beta\right)$ implies $V_a < V_b$. Observing that $V_b \leq 0$ because $\sigma_b = 1$, we may then conclude that $V_a < V_b \leq 0$. Finally, we note that $V_a < 0$ is incompatible with $\sigma_a \in (0,1)$. This concludes the proof. ■

Lemma 11 *There is no partially-mixed equilibrium where* $\sigma_a \in (0,1)$ *and* $\sigma_b = 0$.

Proof. Suppose, by contradiction, that such an equilibrium does exist.

We first show that $\sigma_b = 0$ implies $\gamma_\alpha\,(1-\gamma_\alpha) > \gamma_\beta\left(1-\gamma_\beta\right)$. To establish this, it is sufficient to show that $\left|\gamma_\alpha - \frac{1}{2}\right| < \left|\gamma_\beta - \frac{1}{2}\right|$.

One may readily verify that, for $\sigma_b = 0$, $\gamma_\beta < \frac{1}{2}$. Furthermore, it may be readily shown that $\gamma_\alpha > \frac{1}{2}$ iff

$$\sigma_a > \frac{\frac{1}{2} - (qr + r\,(1-q)\,\rho)}{r\,(1-q)\,(1-\rho)}$$

When $\sigma_a > \frac{\frac{1}{2}-(qr+r(1-q)\rho)}{r(1-q)(1-\rho)}$ (and, hence, $\gamma_\beta < \frac{1}{2}$ and $\gamma_\alpha > \frac{1}{2}$), $\left|\gamma_\alpha - \frac{1}{2}\right| < \left|\gamma_\beta - \frac{1}{2}\right|$ is equivalent to showing that $\gamma_\alpha - \left(1-\gamma_\beta\right) < 0:$

$$\begin{aligned}
\gamma_\alpha - \left(1-\gamma_\beta\right) &= qr + r\,(1-q)\,\rho + r\,(1-q)\,(1-\rho)\,\sigma_a - 1 \\
&\quad + (1-r)\,q + (1-r)\,(1-q)\,\rho + (1-r)\,(1-q)\,(1-\rho)\,\sigma_a \\
&= -(1-q)\,(1-\rho)\,(1-\sigma_a) \\
&< 0
\end{aligned}$$

When $\sigma_a \leq \frac{\frac{1}{2}-(qr+r(1-q)\rho)}{r(1-q)(1-\rho)}$ (and, hence, $\gamma_\beta < \frac{1}{2}$ and $\gamma_\alpha \leq \frac{1}{2}$), $\left|\gamma_\alpha - \frac{1}{2}\right| < \left|\gamma_\beta - \frac{1}{2}\right|$ follows immediately from Lemma **??** which established that $\gamma_\beta < \gamma_\alpha$.

Next we note that $\gamma_\alpha (1-\gamma_\alpha) > \gamma_\beta \left(1-\gamma_\beta\right)$ implies $V_a > V_b$. Observing that $V_b \geq 0$ because $\sigma_b = 0$, we may then conclude that $V_a > V_b \geq 0$. Finally, we note that $V_a > 0$ is incompatible with $\sigma_a \in (0,1)$. This concludes the proof. ■

Proof of Lemma 3:

Recall that q^* is defined as $q^* \equiv \max \left\{ q \in [0,1] \left. \frac{dV_b}{d\sigma_b} \right|_{\sigma_a=0,\sigma_b=1,m=\underline{m}_E(q)} = 0 \right\}$. We prove $q_0 < q^* < q_1$ by showing that:

1. For all $q \leq q_0$, $\left. \frac{dV_b}{d\sigma_b} \right|_{\sigma_a=0,\sigma_b=1} < 0$ for all m.

2. For all $q \geq q_1$, $\left. \frac{dV_b}{d\sigma_b} \right|_{\sigma_a=0,\sigma_b=1} > 0$ for all m.

Existence of q^* then follows from continuity of $\left. \frac{dV_b}{d\sigma_b} \right|_{\sigma_a=0,\sigma_b=1,,m=\underline{m}_E(q)}$ in q and the intermediate value theorem.

To establish 1. and 2., Note that

$$\left. \frac{dV_b}{d\sigma_b} \right|_{\sigma_a=0,\sigma_b=1} = \frac{\Gamma(m+1)}{\Gamma^2\left(\frac{m}{2}+1\right)} \left(\begin{array}{c} r\frac{m}{2}\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{m}{2}-1}\left(1-2\gamma_\beta\right) r (1-q) \rho \\ -(1-r)\frac{m}{2}\left(\gamma_\alpha(1-\gamma_\alpha)\right)^{\frac{m}{2}-1}(1-2\gamma_\alpha)(1-r)(1-q)\rho \end{array} \right)$$

where, since $\sigma_a = 0$ and $\sigma_b = 1$,

$$\begin{aligned} \gamma_\beta &= q(1-r) + (1-q)\rho \\ \gamma_\alpha &= qr + (1-q)\rho \end{aligned}$$

The expression $\left. \frac{dV_b}{d\sigma_b} \right|_{\sigma_a=0,\sigma_b=1}$ takes the sign of

$$r^2 \left(\frac{\gamma_\beta\left(1-\gamma_\beta\right)}{\gamma_\alpha(1-\gamma_\alpha)} \right)^{\frac{m}{2}-1} \left(1-2\gamma_\beta\right) - (1-r)^2 (1-2\gamma_\alpha)$$

To establish 1., notice that for all $q \leq q_0$, $\gamma_\beta\left(1-\gamma_\beta\right) > \gamma_\alpha(1-\gamma_\alpha)$ since $\frac{1}{2} < \gamma_\beta < \gamma_\alpha$ (which, in turn, follows from (1) $\gamma_\beta|_{q=q_1} = \frac{1}{2}$; (2) $q \leq q_0 < q_1$; (3) γ_β is decreasing in q.) Thus,

$$\begin{aligned} & r^2 \left(\frac{\gamma_\beta\left(1-\gamma_\beta\right)}{\gamma_\alpha(1-\gamma_\alpha)} \right)^{\frac{m}{2}-1} \left(1-2\gamma_\beta\right) - (1-r)^2 (1-2\gamma_\alpha) \\ < \; & r^2 \left(1-2\gamma_\beta\right) - (1-r)^2 (1-2\gamma_\alpha) \\ = \; & (2r-1)\left(2q\left(r^2-r+\rho\right)+1-2\rho\right) \end{aligned}$$

Now, notice that $2q(r^2-r+\rho)+1-2\rho \leq 0$ when $q \leq q_0$. Thus, $\left. \frac{dV_b}{d\sigma_b} \right|_{\sigma_a=0,\sigma_b=1} < 0$ for all m.

To establish 2., notice that $\gamma_\alpha > \frac{1}{2}$ and, for all $q \geq q_1$, $\gamma_\beta \leq \frac{1}{2}$. Hence,

$$r^2 \left(\frac{\gamma_\beta (1-\gamma_\beta)}{\gamma_\alpha (1-\gamma_\alpha)} \right)^{\frac{m}{2}-1} (1-2\gamma_\beta) - (1-r)^2 (1-2\gamma_\alpha) > 0$$

This completes the proof.

Proof of Lemma 4:

To prove 1., note that for both kinds of conflicted voters to mix, it must be that σ_a, σ_b are such that

$$\begin{aligned} r \Pr[piv|\alpha] - (1-r) \Pr[piv|\beta] &= \frac{\varepsilon}{1-\varepsilon} \\ r \Pr[piv|\beta] - (1-r) \Pr[piv|\alpha] &= \frac{\varepsilon}{1-\varepsilon} \end{aligned} \tag{8}$$

Hence,

$$\Pr[piv|\alpha] = \Pr[piv|\beta]$$

Substituting this equality back into equation (8) gives

$$\Pr[piv|\alpha] = \Pr[piv|\beta] = \frac{1}{2r-1} \frac{\varepsilon}{1-\varepsilon}$$

To prove 2. and 3., recall that

$$\Pr[piv|\theta] = \binom{n}{\frac{n}{2}} (\gamma_\theta)^{\frac{n}{2}} (1-\gamma_\theta)^{\frac{n}{2}}$$

From the fact that $\Pr[piv|\alpha] = \Pr[piv|\beta]$ it then follows that

$$\binom{n}{\frac{n}{2}} (\gamma_\alpha)^{\frac{n}{2}} (1-\gamma_\alpha)^{\frac{n}{2}} = \binom{n}{\frac{n}{2}} (\gamma_\beta)^{\frac{n}{2}} (1-\gamma_\beta)^{\frac{n}{2}}$$

Hence, either $\gamma_\alpha = \gamma_\beta$ or $\gamma_\alpha = 1-\gamma_\beta$.

To see that the $\gamma_\alpha = \gamma_\beta$ is impossible, recall that

$$\begin{aligned} \gamma_\alpha &= qr + r(1-q)\rho + r(1-q)(1-\rho)\sigma_a + (1-r)(1-q)\rho\sigma_b \\ \gamma_\beta &= (1-r)q + (1-r)(1-q)\rho + (1-r)(1-q)(1-\rho)\sigma_a + r(1-q)\rho\sigma_b \end{aligned}$$

Differencing these expressions yields

$$\begin{aligned} \gamma_\alpha - \gamma_\beta &= (2r-1)(q + \rho(1-q) + (1-q)(1-\rho)\sigma_a + (1-q)\rho\sigma_b) \\ &> 0 \end{aligned}$$

for all $\sigma_a, \sigma_b \in (0,1)$. Hence, $\gamma_\alpha = 1-\gamma_\beta$

Finally, $\gamma_\alpha = 1 - \gamma_\beta$ implies that

$$\sigma_a = 1 - \frac{\rho}{1-\rho}\sigma_b$$

This completes the proof.

Proof of Proposition 4:
In a completely-mixed equilibrium,

$$\begin{aligned} V_a &\equiv \binom{n}{\frac{n}{2}}\left(r\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}} - \left(1-r\right)\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}\right) - \frac{\varepsilon}{1-\varepsilon} = 0 \\ V_b &\equiv \binom{n}{\frac{n}{2}}\left(r\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}} - \left(1-r\right)\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}\right) - \frac{\varepsilon}{1-\varepsilon} = 0 \end{aligned}$$

Using the fact that by Lemma 4, $\gamma_\alpha = 1 - \gamma_\beta$, these two equations reduce to

$$\binom{n}{\frac{n}{2}}\left(\left(2r-1\right)\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}}\right) - \frac{\varepsilon}{1-\varepsilon} = 0$$

Rewriting reveals

$$\gamma_\alpha\left(1-\gamma_\alpha\right) = \left(\frac{\varepsilon}{1-\varepsilon}\frac{1}{\left(2r-1\right)\binom{n}{\frac{n}{2}}}\right)^{\frac{2}{n}} = K\left(n\right) \tag{9}$$

Hence, we have to show that this equation has a solution in γ_α that is feasible—i.e., that this γ_α can be attainted for some combination of σ_a, $\sigma_b \in (0,1)$—iff $\bar{m}_I < n < \bar{m}_{CM}$. What is the feasible range for γ_α?

From Lemma 4 we know that $\sigma_a = 1 - \frac{\rho}{1-\rho}\sigma_b$ over the range $\sigma_b \in \left(0, \frac{1-\rho}{\rho}\right)$. Hence, the feasible range for γ_α is

$$\begin{aligned} \gamma_\alpha &\in \left(r - \left(2r-1\right)\left(1-q\right)\left(1-\rho\right), r\right) \\ &= \left(\hat{\gamma}_\alpha, r\right) \end{aligned}$$

where it is easily verified that $\hat{\gamma}_\alpha \geq \frac{1}{2}$.

We may conclude that for equation (9) to have a feasible solution in γ_a it must be that $K\left(n\right) \in \left(r\left(1-r\right), \hat{\gamma}_\alpha\left(1-\hat{\gamma}_\alpha\right)\right)$. Hence, to prove the first part of the proposition, it remains to be shown that $K\left(n\right) \in \left(r\left(1-r\right), \hat{\gamma}_\alpha\left(1-\hat{\gamma}_\alpha\right)\right)$ iff $\bar{m}_I < n < \bar{m}_{CM}$.

We first show that $K\left(n\right) > r\left(1-r\right)$ iff $n > \bar{m}_I$. Note that $K\left(n\right) > r\left(1-r\right)$ is equivalent to

$$\binom{n}{\frac{n}{2}}\left(2r-1\right)\left(r\left(1-r\right)\right)^{\frac{n}{2}} < \frac{\varepsilon}{1-\varepsilon}$$

which is exactly the condition for a purely instrumental equilibrium *not* to exist. Proposition 1 then implies that this is the case iff $n > \bar{m}_I$.

Next we show that $K(n) < \hat{\gamma}_\alpha (1 - \hat{\gamma}_\alpha)$ iff $n < \bar{m}_{CM}$. Note that $K(n) \leq \hat{\gamma}_\alpha (1 - \hat{\gamma}_\alpha)$ is equivalent to

$$\binom{n}{\frac{n}{2}} (2r - 1) (\hat{\gamma}_\alpha (1 - \hat{\gamma}_\alpha))^{\frac{n}{2}} \geq \frac{\varepsilon}{1 - \varepsilon}$$

By definition, at $\bar{m}_{CM}$, the continuous analogue of the above inequality hold with equality. Moreover, the left-hand side of the above expression is a special case of Lemma 2, with $z_\alpha = z_\beta = \hat{\gamma}_\alpha (1 - \hat{\gamma}_\alpha)$. Thus, we know that $\binom{n}{\frac{n}{2}} (2r - 1) (\hat{\gamma}_\alpha (1 - \hat{\gamma}_\alpha))^{\frac{n}{2}}$ is strictly decreasing in n and, hence, we may conclude that $K(n) < \hat{\gamma}_\alpha (1 - \hat{\gamma}_\alpha)$ iff $n < \bar{m}_{CM}$. This completes the proof of the first part of the proposition.

Finally, we establish that $\bar{m}_{CM} > \bar{m}_I$. Notice that $\gamma_\alpha < r$, since $(r\rho + (1 - r)(1 - \rho)) < r$. Hence, $\frac{1}{2} < \hat{\gamma}_\alpha < r$ and, therefore, $r(1 - r) < \gamma_\alpha (1 - \gamma_\alpha)$. As a result

$$\binom{n}{\frac{n}{2}} (2r - 1) (\hat{\gamma}_\alpha (1 - \hat{\gamma}_\alpha))^{\frac{n}{2}} > \binom{n}{\frac{n}{2}} (2r - 1) (r (1 - r))^{\frac{n}{2}}$$

and the claim follows.

Proof of Proposition 5:

Recall that any completely-mixed equilibrium is characterized by a feasible value of γ_α that solves

$$\binom{n}{\frac{n}{2}} \left((2r - 1) (\gamma_\alpha (1 - \gamma_\alpha))^{\frac{n}{2}} \right) = \frac{\varepsilon}{1 - \varepsilon} \tag{10}$$

By the Lemma 2, the LHS of this expression is decreasing in n. Furthermore, for all feasible solutions, $\gamma_\alpha > \frac{1}{2}$. As a consequence, any value of γ_α solving equation (10) is decreasing in n. Moreover, because $\gamma_\beta = 1 - \gamma_\alpha < \frac{1}{2}$, γ_β is increasing.

Next, recall that, in any completely-mixed equilibrium, we have $\sigma_b = \frac{1-\rho}{\rho} (1 - \sigma_a)$. Hence, σ_a and σ_b move in opposite directions in n.

Furthermore,

$$\begin{aligned} \gamma_\alpha &= qr + r(1 - q)\rho + r(1 - q)(1 - \rho)\sigma_a + (1 - r)(1 - q)\rho\sigma_b \\ &= qr + r(1 - q)\rho + r(1 - q)(1 - \rho)\sigma_a + (1 - r)(1 - q)(1 - \rho)(1 - \sigma_a) \end{aligned}$$

such that

$$\begin{aligned} \frac{d\gamma_\alpha}{d\sigma_a} &= r(1 - q)(1 - \rho) - (1 - r)(1 - q)(1 - \rho) \\ &= (2r - 1)(1 - q)(1 - \rho) \\ &> 0 \end{aligned}$$

Hence, σ_a is decreasing and σ_b is increasing in n in any completely-mixed equilibrium. This completes the proof.

Proof of Proposition 6:

By Lemma 12 (below), in any partially-mixed equilibrium, $\sigma_b \in [\frac{1-\rho}{\rho}, 1)$.

Next, we claim that $V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} < 0$ iff $n > \bar{m}_{CM}$: By construction, at $\bar{m}_{CM}$, $V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} = 0$. From Lemma 2, it follows that $V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}}$ is strictly decreasing in n and hence $V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} < 0$ iff $n > \bar{m}_{CM}$.

We also claim that $V_b|_{\sigma_a=0,\sigma_b=1} > 0$ iff $n < \underline{m}_E$: By construction, at $\underline{m}_E$, $V_b|_{\sigma_a=0,\sigma_b=1} = 0$. From Lemma 2, it follows that $V_b|_{\sigma_a=0,\sigma_b=1}$ is strictly decreasing in n, hence, $V_b|_{\sigma_a=0,\sigma_b=1} > 0$ iff $n \geq \underline{n}_E$.

From Lemma 14 (below), which shows that under high correlation V_b is strictly increasing in $\sigma_b \in [\frac{1-\rho}{\rho}, 1)$, it then follows that for all $\bar{m}_{CM} \leq n < \underline{m}_E$ there exists a unique value $\sigma_b \in \left(\frac{1-\rho}{\rho}, 1\right)$ where $V_b = 0$. It is straightforward to verify that, at this value of σ_b, $V_a < 0$; hence this comprises a partially-mixed equilibrium.

Finally, we establish that $\bar{m}_{CM} < \underline{m}_E$. At $\bar{m}_{CM}$, $V_b|_{\sigma_b=\frac{1-\rho}{\rho}} = 0$. Lemma 14 implies that, at $\bar{m}_{CM}$, $V_b|_{\sigma_b=1} > 0$. Moreover, from Lemma 2 we know that $V_b|_{\sigma_b=1}$ is strictly decreasing in m. Because, at $\underline{m}_E$, $V_b|_{\sigma_b=\frac{1-\rho}{\rho}} = 0$, this implies that $\underline{m}_E > \bar{m}_{CM}$.

Lemma 12 *In any partially-mixed equilibrium,* $\sigma_b \geq \frac{1-\rho}{\rho}$.

Proof. We will prove the lemma by showing that $\sigma_b < \frac{1-\rho}{\rho}$ implies $\gamma_\alpha (1-\gamma_\alpha) > \gamma_\beta \left(1-\gamma_\beta\right)$, which contradicts Lemma 13 (below). Hence, $\sigma_b \geq \frac{1-\rho}{\rho}$.

Recall that in any partially-mixed equilibrium $\sigma_a = 0$ and $\sigma_b \in (0,1)$. Hence,

$$\begin{aligned}\gamma_\alpha &= qr + r(1-q)\rho + (1-r)(1-q)\rho\sigma_b \\ \gamma_\beta &= (1-r)q + (1-r)(1-q)\rho + r(1-q)\rho\sigma_b\end{aligned}$$

It may be readily verified that $\gamma_\alpha > \gamma_\beta$.

First, we find the value of σ_b that makes $\gamma_\alpha = \frac{1}{2}$. This may readily be shown to be

$$\sigma_b^{\gamma_\alpha=\frac{1}{2}} = \frac{1-2qr-2r(1-q)\rho}{2(1-r)(1-q)\rho}$$

We claim that $\sigma_b^{\gamma_\alpha=\frac{1}{2}} < \frac{1-\rho}{\rho}$. To see this, notice that

$$\begin{aligned}& \frac{1-2qr-2r(1-q)\rho}{2(1-r)(1-q)\rho} - \frac{1-\rho}{\rho} \\ =& \frac{1-2qr-2r(1-q)\rho-2(1-r)(1-q)(1-\rho)}{2(1-r)(1-q)\rho^2} \\ =& -\frac{(2r-1)(2q(1-\rho)+2\rho-1)}{2(1-r)(1-q)\rho^2} \\ <& \; 0\end{aligned}$$

Thus, for $\sigma_b \leq \sigma_b^{\gamma_\alpha=\frac{1}{2}}$, $\gamma_\beta < \gamma_\alpha \leq \frac{1}{2}$, and hence $\gamma_\alpha (1-\gamma_\alpha) > \gamma_\beta \left(1-\gamma_\beta\right)$.

Next, we find the value of σ_b that makes $\gamma_\beta = \frac{1}{2}$. This may be readily shown to be

$$\sigma_b^{\gamma_\beta=\frac{1}{2}} = \frac{1-2qr-2r(1-q)\rho}{2(1-r)(1-q)\rho}$$

We claim that $\sigma_b^{\gamma_\beta=\frac{1}{2}} > \frac{1-\rho}{\rho}$. To see this, notice that

$$\begin{aligned} & \frac{1-2(1-r)q-2(1-r)(1-q)\rho}{2r(1-q)\rho} - \frac{1-\rho}{\rho} \\ = & \frac{1-2(1-r)q-2(1-r)(1-q)\rho-(1-\rho)2r(1-q)}{2r(1-q)\rho^2} \\ = & \frac{(2r-1)(2q(1-\rho)+2\rho-1)}{2r(1-q)\rho^2} \\ > & \ 0 \end{aligned}$$

In the region $\sigma_b^{\gamma_\alpha=\frac{1}{2}} < \sigma_b < \sigma_b^{\gamma_\beta=\frac{1}{2}}$, we know that $\gamma_\alpha(1-\gamma_\alpha) - \gamma_\beta\left(1-\gamma_\beta\right)$ is strictly decreasing. Next, notice that, at $\sigma_b = \frac{1-\rho}{\rho}$, $\gamma_\alpha = 1-\gamma_\beta$ and hence, at that value $\gamma_\alpha(1-\gamma_\alpha) = \gamma_\beta\left(1-\gamma_\beta\right)$. Hence, we may conclude that for all $\sigma_b < \frac{1-\rho}{\rho}$, $\gamma_\alpha(1-\gamma_\alpha) > \gamma_\beta\left(1-\gamma_\beta\right)$. And, as shown in Lemma 13, this is incompatible with a partially-mixed equilibrium. Hence, it must be that $\sigma_b \geq \frac{1-\rho}{\rho}$. ∎

Lemma 13 *In any partially-mixed equilibrium,* $\gamma_\alpha(1-\gamma_\alpha) \leq \gamma_\beta\left(1-\gamma_\beta\right)$.

Proof. In any partially-mixed equilibrium,

$$\begin{aligned} V_b &= r\binom{n}{\frac{n}{2}}\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}} - (1-r)\binom{n}{\frac{n}{2}}\left(\gamma_\alpha(1-\gamma_\alpha)\right)^{\frac{n}{2}} - \frac{\varepsilon}{1-\varepsilon} = 0 \\ V_a &= r\binom{n}{\frac{n}{2}}\left(\gamma_\alpha(1-\gamma_\alpha)\right)^{\frac{n}{2}} - (1-r)\binom{n}{\frac{n}{2}}\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}} - \frac{\varepsilon}{1-\varepsilon} \leq 0 \end{aligned}$$

Together, these two conditions imply that

$$\begin{aligned} & r\binom{n}{\frac{n}{2}}\left(\gamma_\alpha(1-\gamma_\alpha)\right)^{\frac{n}{2}} - (1-r)\binom{n}{\frac{n}{2}}\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}} \\ & -\left(r\binom{n}{\frac{n}{2}}\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}} - (1-r)\binom{n}{\frac{n}{2}}\left(\gamma_\alpha(1-\gamma_\alpha)\right)^{\frac{n}{2}}\right) \\ \leq & \ 0 \end{aligned}$$

And this inequality holds iff

$$\gamma_\alpha(1-\gamma_\alpha) \leq \gamma_\beta\left(1-\gamma_\beta\right)$$

That completes the proof. ∎

Lemma 14 *For $q \geq q_1$, $V_b|_{\sigma_a=0}$ is strictly increasing in $\sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]$.*

Proof. Differentiating V_b with respect to σ_b yields

$$\left.\frac{\partial V_b}{\partial \sigma_b}\right|_{\sigma_a=0} = \binom{n}{\frac{n}{2}} \frac{n}{2} (1-q) \rho \left(\begin{array}{c} \left(\gamma_\beta \left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1} \left(1-2\gamma_\beta\right) r^2 \\ -\left(\gamma_\alpha \left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1} \left(1-2\gamma_\alpha\right) (1-r)^2 \end{array} \right)$$

which takes the sign of

$$\left(r^2 \left(\gamma_\beta \left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1} \left(1-2\gamma_\beta\right) - (1-r)^2 \left(\gamma_\alpha \left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1} \left(1-2\gamma_\alpha\right)\right)$$

And this expression is strictly positive because $\gamma_\alpha > \frac{1}{2}$, while, for $q \geq q_1 = \frac{1}{2}\frac{2\rho-1}{\rho-(1-r)}$, $\gamma_\beta|_{\sigma_a=0} \leq \frac{1}{2}$ for all $\sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]$. ■

Proof of Proposition 7:

In a partially-mixed equilibrium, σ_b solves $V_b = 0$. From Lemma 2, for any fixed value of σ_b, V_b is strictly decreasing in n. Furthermore, for any fixed n, from Lemma 14, we know that V_b is strictly increasing in σ_b. Together these two facts imply that σ_b is increasing in n in the partially-mixed equilibrium sequence. As σ_a remains constant at zero, voting becomes more expressive when n increases.

Proof of Lemma 5:

Recall that $\bar{m}_{PM} \equiv \max\left\{m |\ V_b|_{\sigma_a=0} = 0 \text{ has a solution in } \sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]\right\}$.

By Lemma 15 (below), for $q \leq q_0$, $V_b|_{\sigma_a=0}$ is single-peaked in σ_b on the interval $\left[\frac{1-\rho}{\rho}, 1\right]$, while $\frac{dV_b}{d\sigma_b}|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} > 0$ and $\frac{dV_b}{d\sigma_b}|_{\sigma_a=0,\sigma_b=1} < 0$ for all m. Hence, the unique σ_b that maximizes $V_b|_{\sigma_a=0}$ over the interval $\left[\frac{1-\rho}{\rho}, 1\right]$ lies strictly in the interior of that interval. Denote this σ_b by σ_b'. By the envelope theorem,

$$\begin{aligned} \frac{d}{dm} V_b|_{\sigma_a=0,\sigma_b=\sigma_b'(m)} &= \left.\frac{\partial \left(V_b|_{\sigma_a=0}\right)}{\partial m}\right|_{\sigma_b=\sigma_b'(m)} + \left.\frac{\partial \left(V_b|_{\sigma_a=0}\right)}{\partial \sigma_b}\right|_{\sigma_b=\sigma_b'(m)} \frac{\partial \sigma_b'(m)}{\partial m} \\ &= \left.\frac{\partial \left(V_b|_{\sigma_a=0}\right)}{\partial m}\right|_{\sigma_b=\sigma_b'(m)} \end{aligned}$$

For $\sigma_a = 0$ and $\sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]$, $\gamma_\beta \left(1-\gamma_\beta\right) > \gamma_\alpha \left(1-\gamma_\alpha\right)$ by Lemma 8. Hence, Lemma 2 implies that $\left.\frac{\partial (V_b|_{\sigma_a=0})}{\partial m}\right|_{\sigma_b=\sigma_b'(m)}$ (and, therefore, $\frac{d}{dm}V_b|_{\sigma_a=0,\sigma_b=\sigma_b'(m)}$) is strictly negative. From here, the proof of existence and uniqueness proceeds analogous to that for $\underline{m}_E$ in the main text.

To prove that $\bar{m}_{PM} > \underline{m}_E$, note that, at $m = \underline{m}_E$, $V_b|_{\sigma_a=0,\sigma_b=1} = 0$. By Lemma 15 we know that $\frac{dV_b}{d\sigma_b}|_{\sigma_a=0,\sigma_b=1} < 0$. Hence, for some σ_b' strictly smaller than but close to 1, $V_b|_{\sigma_a=0,\sigma_b=\sigma_b'} > 0$. Lemma 2 then implies that there exists an $m > \underline{m}_E$ such that the equation $V_b|_{\sigma_a=0} = 0$ has a solution in $\sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]$. Therefore, $\bar{m}_{PM}$, which is defined as the largest m for which such a solution exists, must also be strictly greater than $\underline{m}_E$.

Lemma 15 *For $q \leq q_0$, $V_b|_{\sigma_a=0}$ is single-peaked in σ_b on the interval $\left[\frac{1-\rho}{\rho}, 1\right]$. Moreover $\left.\frac{dV_b}{d\sigma_b}\right|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} > 0$ and $\left.\frac{dV_b}{d\sigma_b}\right|_{\sigma_a=0,\sigma_b=1} < 0$ for all m.*

Proof. To prove single-peakedness of $V_b|_{\sigma_a=0}$, we show that $\left.\frac{dV_b}{d\sigma_b}\right|_{\sigma_a=0}$ satisfies single-crossing of zero in σ_b, for $\sigma_b \in \left[\frac{1-\rho}{\rho}, 1\right]$.

Recall that

$$\left.\frac{dV_b}{d\sigma_b}\right|_{\sigma_a=0} = \binom{n}{\frac{n}{2}} \begin{pmatrix} r\frac{n}{2}\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\beta\right)r\left(1-q\right)\rho \\ -\left(1-r\right)\frac{n}{2}\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\alpha\right)\left(1-r\right)\left(1-q\right)\rho \end{pmatrix}$$

Hence, $\left.\frac{dV_b}{d\sigma_b}\right|_{\sigma_a=0}$ is proportional to

$$r^2\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\beta\right) - \left(1-r\right)^2\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\alpha\right)$$

and hence, at a crossing point,

$$r^2\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\beta\right) = \left(1-r\right)^2\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\alpha\right) \tag{11}$$

or

$$\begin{aligned} r^2\left(\frac{\gamma_\beta\left(1-\gamma_\beta\right)}{\gamma_\alpha\left(1-\gamma_\alpha\right)}\right)^{\frac{n}{2}-1}\left(1-2\gamma_\beta\right) &= \left(1-r\right)^2\left(1-2\gamma_\alpha\right) \\ \left(\frac{\gamma_\beta\left(1-\gamma_\beta\right)}{\gamma_\alpha\left(1-\gamma_\alpha\right)}\right)^{\frac{n}{2}-1} &= \frac{\left(1-r\right)^2}{r^2}\frac{\left(1-2\gamma_\alpha\right)}{\left(1-2\gamma_\beta\right)} \end{aligned}$$

Next, notice that $\left.\frac{d^2V_b}{\left(d\sigma_b\right)^2}\right|_{\sigma_a=0}$ is proportional to

$$\begin{aligned} &r^3\left(\frac{n}{2}-1\right)\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-2}\left(1-2\gamma_\beta\right)^2\left(1-q\right)\rho - 2r^3\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1}\left(1-q\right)\rho \\ &-\left(1-r\right)^3\left(\frac{n}{2}-1\right)\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-2}\left(1-2\gamma_\alpha\right)^2\left(1-q\right)\rho \\ &+2\left(1-r\right)^3\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1}\left(1-q\right)\rho \end{aligned}$$

which, in turn, is proportional to

$$\left(\frac{n}{2}-1\right)\left(r^3\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-2}\left(1-2\gamma_\beta\right)^2-(1-r)^3\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-2}\left(1-2\gamma_\alpha\right)^2\right)$$
$$-2r^3\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1}+2\left(1-r\right)^3\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1}$$

Rewriting,

$$\left(\frac{n}{2}-1\right)\left(\begin{array}{c} r^3\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-2}\left(1-2\gamma_\beta\right)^2 \\ -\left(1-r\right)\frac{(1-2\gamma_\alpha)}{\gamma_\alpha(1-\gamma_\alpha)}\left(1-r\right)^2\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\alpha\right)\end{array}\right)$$
$$-2r^2\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1}+2\left(1-r\right)^2\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1}$$

Using the the fact that we are at a crossing point,

$$\left(\frac{n}{2}-1\right)\left(\begin{array}{c} r^3\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-2}\left(1-2\gamma_\beta\right)^2 \\ -\left(1-r\right)\frac{(1-2\gamma_\alpha)}{\gamma_\alpha(1-\gamma_\alpha)}r^2\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\beta\right)\end{array}\right)$$
$$-2r^2\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1}+2\left(1-r\right)^2\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1}$$

Now note that the sum of the last two terms is for sure negative because $r>\frac{1}{2}$ and, in the area of interest, $\frac{1}{2}<\gamma_\beta<\gamma_\alpha$. Hence, the question reduces to whether at a crossing point

$$\frac{\gamma_\alpha\left(1-\gamma_\alpha\right)}{\gamma_\beta\left(1-\gamma_\beta\right)}<\frac{1-r}{r}\frac{1-2\gamma_\alpha}{1-2\gamma_\beta}$$

At crossing:

$$\frac{\left(1-2\gamma_\alpha\right)}{\left(1-2\gamma_\beta\right)}=\left(\frac{\gamma_\beta\left(1-\gamma_\beta\right)}{\gamma_\alpha\left(1-\gamma_\alpha\right)}\right)^{\frac{n}{2}-1}\frac{r^2}{\left(1-r\right)^2}$$

Subbing this into the inequality, we get

$$\frac{\gamma_\alpha\left(1-\gamma_\alpha\right)}{\gamma_\beta\left(1-\gamma_\beta\right)}<\left(\frac{\gamma_\beta\left(1-\gamma_\beta\right)}{\gamma_\alpha\left(1-\gamma_\alpha\right)}\right)^{\frac{n}{2}-1}\frac{r}{1-r}$$

And this is true for all n, because because $r>\frac{1}{2}$ and, in the area of interest, $\frac{1}{2}<\gamma_\beta<\gamma_\alpha$, such that $\gamma_\alpha\left(1-\gamma_\alpha\right)<\gamma_\beta\left(1-\gamma_\beta\right)$. This completes the proof of single-peakedness.

To see that $\left.\frac{dV_b}{d\sigma_b}\right|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}}>0$, we differentiate V_b with respect to σ_b, which yields

$$\frac{dV_b}{d\sigma_b}=\binom{n}{\frac{n}{2}}\frac{n}{2}\left(1-q\right)\rho\left(\begin{array}{c}\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\beta\right)r^2 \\ -\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\alpha\right)\left(1-r\right)^2\end{array}\right)$$

and takes the sign of

$$\left(r^2\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\beta\right)-(1-r)^2\left(\gamma_\alpha\left(1-\gamma_\alpha\right)\right)^{\frac{n}{2}-1}\left(1-2\gamma_\alpha\right)\right)$$

At $\sigma_a = 0$ and $\sigma_b = \frac{1-\rho}{\rho}$, this expression is strictly positive because $\gamma_\alpha > \frac{1}{2}$ and $\gamma_\beta < \frac{1}{2}$, where the latter follows from

$$\begin{aligned}\gamma_\beta\big|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} &= q(1-r)+(1-q)\left((1-r)\rho+r(1-\rho)\right)\\ &< q(1-r)+(1-q)\left((1-r)\frac{1}{2}+r\frac{1}{2}\right)\\ &= q(1-r)+(1-q)\frac{1}{2}\\ &\leq \frac{1}{2}\end{aligned}$$

Finally, the argument why $\frac{dV_b}{d\sigma_b}\Big|_{\sigma_a=0,\sigma_b=1} < 0$ can be found in the proof of Lemma 3. ■

Proof of Proposition 10:

The proofs of parts (1), (2), and (4) are identical to those of Propositions 1, 4, 2, respectively. It remains to that: 1) Low partially-mixed voting is an equilibrium iff $\bar{m}_{CM} \leq n < \bar{m}_{PM}$, 2) high partially-mixed voting is an equilibrium iff $\underline{m}_E \leq n < \bar{m}_{PM}$, 3) there are no other partially-mixed equilibria.

In a partially-mixed equilibrium, $\sigma_a = 0$, $\sigma_b \in (0,1)$. Moreover, as Lemma 12 continues to hold for $q \leq q_0$, we know that, in fact, $\sigma_b \in [\frac{1-\rho}{\rho}, 1)$.

Also unchanged from the high correlation case remain the arguments as to why $V_b|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} < 0$ iff $n > \bar{m}_{CM}$, and $V_b|_{\sigma_a=0,\sigma_b=1} < 0$ iff $n > \underline{m}_E$. (See proof of Proposition 6.). Moreover, we claim that $V_b|_{\sigma_a=0,\sigma_b=\sigma_{b,\bar{m}_{PM}}} < 0$ iff $n > \bar{m}_{PM}$: By construction, at $\bar{m}_{PM}$, $V_b|_{\sigma_a=0,\sigma_b=\sigma_{b,\bar{m}_{PM}}} = 0$. From Lemma 2, it follows that $V_b|_{\sigma_a=0,\sigma_b=\sigma_{big}}$ is strictly decreasing in n. Hence, $V_b|_{\sigma_a=0,\sigma_b=\sigma_{b,\bar{m}_{PM}}} < 0$ iff $n > \bar{m}_{PM}$.

Next, recall that Lemma 15 establishes that, for $q \leq q_0$, $V_b|_{\sigma_a=0}$ is single peaked in σ_b on the interval $\left[\frac{1-\rho}{\rho}, 1\right]$, while $\frac{dV_b}{d\sigma_b}|_{\sigma_a=0,\sigma_b=\frac{1-\rho}{\rho}} > 0$ and $\frac{dV_b}{d\sigma_b}|_{\sigma_a=0,\sigma_b=1} < 0$ for all m.

Combining the facts above implies that there exists a unique $\sigma_b^{LPM}(n) \in \left[\frac{1-\rho}{\rho}, \sigma_b'(m)\right]$ such that $V_b|_{\sigma_a=0,\sigma_b=\sigma_b^{LPM}(n)} = 0$ iff $\bar{m}_{PM} \leq n \leq \bar{m}_{PM}$. Similarly, there exists a unique $\sigma_b^{HPM}(n) \in [\sigma_b'(m), 1]$ such that $V_b|_{\sigma_a=0,\sigma_b=\sigma_b^{HPM}(n)} = 0$ iff $\underline{m}_E < n \leq \bar{m}_{PM}$. By Lemma 2, σ_b^{LPM} must be strictly increasing in n and, ignoring integer constraints, move from $\sigma_b^{LPM} = \frac{1-\rho}{\rho}$ at $\bar{m}_{PM}$, to $\sigma_b^{LPM} = \sigma_{b,\bar{m}_{PM}}$ at $\bar{m}_{PM}$. For the same reason, $\sigma_b^{HPM}(n)$ must be decreasing in n and, again ignoring integer constraints, move from $\sigma_b^{HPM} = 1$ at $\underline{m}_E$ to $\sigma_b^{HPM} = \sigma_{b,\bar{m}_{PM}}$ at $\bar{m}_{PM}$. Note that, therefore, $\sigma_b^{LPM}(n)$ constitutes a low partially-mixed equilibrium sequence, while $\sigma_b^{HPM}(n)$ constitutes a high partially-mixed equilibrium sequence.

Finally, single-peakedness of $V_b|_{\sigma_a=0}$ in σ_b on the interval $\left[\frac{1-\rho}{\rho}, 1\right]$ implies that there can be no other partially-mixed equilibria than the low and high partially-mixed equilibrium sequences that we have just identified.

Proof of Proposition 6:
Instrumental versus high partially-mixed voting.

First, we will show that, in state β, instrumental accuracy is greater than expressive accuracy, i.e., $1-\gamma_{\beta,I} > 1-\gamma_{\beta,HPM}$. This amounts to showing that

$$r > 1 - q(1-r) - (1-q)\rho(1-r+r\sigma_b)$$

or, equivalently,

$$1 - r < q(1-r) + (1-q)\rho(1-r+r\sigma_b)$$

Next, notice that the RHS is at least

$$q(1-r) + (1-q)\rho\left(1-r+r\frac{1-\rho}{\rho}\right)$$

Which reduces to

$$\begin{aligned} & q(1-r) + (1-q)\rho(1-r) + (1-q)r(1-\rho) \\ > \; & q(1-r) + (1-q)\rho(1-r) + (1-q)(1-r)(1-\rho) \\ = \; & 1-r \end{aligned}$$

and this completes the argument.

Next, if $r > qr + (1-q)\rho(r+(1-r)\sigma_b)$, then $\gamma_{\alpha,I} \geq \gamma_{\alpha,HPM}$. Hence, in both states of the world, the election gets less accurate if we move from instrumental to HPM voting. Hence, instrumental voting dominates: $S_I > S_{HPM}$. If $r < qr + (1-q)\rho(r+(1-r)\sigma_b)$, then $\gamma_{\alpha,HPM} > \gamma_{\alpha,I}$. Hence, when we move from instrumental to high partially-mixed voting, the election gets more accurate in state α, but less accurate in state state β. Hence, there is a trade-off.

The marginal effect on S of a fall in $1-\gamma_\beta$ is proportional to $\left(\gamma_\beta\left(1-\gamma_\beta\right)\right)^{\frac{n}{2}}$, while the marginal effect on S of an increase in γ_α is proportional to $(\gamma_\alpha(1-\gamma_\alpha))^{\frac{n}{2}}$. Note that $\gamma_{\alpha,I} = 1-\gamma_{\beta,I} = r$. Hence, when we start increasing γ_α from $\gamma_{\alpha,I}$ towards $\gamma_{\alpha,HPM}$ and, simultaneously, decreasing $1-\gamma_\beta$ from $1-\gamma_{\beta,I}$ to $1-\gamma_{\beta,HPM}$, initially, the marginal effects on S exactly cancel each other. However, as $r > \frac{1}{2}$, increasing γ_α means that $\gamma_\alpha(1-\gamma_\alpha)$ decreases, and, hence, the subsequent positive marginal effect on S decreases. At the same time, decreasing $1-\gamma_\beta$ means that, at least initially, $\gamma_\beta\left(1-\gamma_\beta\right)$ increases and, hence, the subsequent negative marginal effect on S increases. This implies that, all along the path from $\left(\gamma_{\alpha,I}, 1-\gamma_{\beta,I}\right)$ to $\left(\gamma_{\alpha,HPM}, 1-\gamma_{\beta,HPM}\right)$, $\gamma_\beta\left(1-\gamma_\beta\right) > \gamma_\alpha(1-\gamma_\alpha)$. Hence, we may conclude that the negative effect on S of the decrease in $1-\gamma_\beta$ dominates the positive effect of the increase in γ_α both in size and in impact, such that $S_I > S_{HPM}$.

High partially-mixed versus expressive voting.

Note that $\gamma_{\alpha,HPM} < \gamma_{\alpha,E}$, while $1 - \gamma_{\beta,HPM} > 1 - \gamma_{\beta,E}$. Hence, when we move from HPM to expressive voting, the election gets more accurate in state α, but less accurate in state state β. Hence, there is a trade-off.

The increase in accuracy in state α is

$$\gamma_{\alpha,E} - \gamma_{\alpha,HPM} = (1-r)(1-q)\rho(1-\sigma_b)$$

which is strictly smaller than the decrease in accuracy in state β, which is

$$\left(1 - \gamma_{\beta,HPM}\right) - \left(1 - \gamma_{\beta,E}\right) = r(1-q)\rho(1-\sigma_b)$$

Moreover, the effect on S of a decrease in $\left(1-\gamma_\beta\right)$ is stronger than the effect of even a same-size increase in γ_a. To establish this, it is sufficient to show that $\left|\frac{1}{2} - \gamma_\alpha\right| > \left|\frac{1}{2} - \left(1-\gamma_\beta\right)\right|$, such that $\gamma_\beta\left(1-\gamma_\beta\right) > \gamma_\alpha(1-\gamma_\alpha)$.

$\left|\frac{1}{2} - \gamma_\alpha\right| > \left|\frac{1}{2} - \left(1-\gamma_\beta\right)\right|$ is equivalent to

$$\begin{aligned} & \left|qr + (1-q)\rho + (1-r)(1-q)\rho\sigma_b - \frac{1}{2}\right| \\ > & \left|q(1-r) + (1-r)(1-q)\rho + r(1-q)\rho\sigma_b - \frac{1}{2}\right| \end{aligned}$$

If $qr + (1-q)\rho + (1-r)(1-q)\rho\sigma_b - \frac{1}{2} \geq 0$, the inequality becomes

$$(r - (1-r))(q + (1-q)\rho(1-\sigma_b)) > 0$$

which is indeed true.

If $qr + (1-q)\rho + (1-r)(1-q)\rho\sigma_b - \frac{1}{2} < 0$, the inequality becomes

$$(2\rho - 1 + 2\rho\sigma_b)(1-q) > 0$$

which also holds.

Hence, we may indeed conclude that $S_{HPM} > S_E$.

Low partially-mixed versus high partially-mixed voting

The only difference between LPM and HPM voting is that $\sigma_{b,LPM} < \sigma_{b,HPM}$. Hence, in moving from a LPM to a HPM equilibrium, the election gets more accurate in state α, but less accurate in state state β. Hence, there is a trade-off.

Note that the increase in accuracy in state α is

$$\gamma_{\alpha,HPM} - \gamma_{\alpha,LPM} = (1-r)(1-q)\rho(\sigma_{b,HPM} - \sigma_{b,LPM})$$

which is smaller than the decrease in accuracy in state β, which is

$$\left(1 - \gamma_{\beta,LPM}\right) - \left(1 - \gamma_{\beta,HPM}\right) = r(1-q)\rho(\sigma_{b,HPM} - \sigma_{b,LPM})$$

Moreover, along the same lines as above, it can easily be shown that also the impact on S of an increase in accuracy in state α is smaller than the impact of even a same-size increase in the accuracy in state β. Hence, $S_{LPM} > S_{HPM}$.

Completely mixed versus high partially-mixed voting.

The difference in success probabilities in state α is

$$\gamma_{\alpha,CM} - \gamma_{\alpha,HPM} = r\left(1-q\right)\left(1-\rho\right)\sigma_{a,CM} - \left(1-r\right)\left(1-q\right)\rho\left(\sigma_{b,HPM} - \sigma_{b,CM}\right)$$

Using $\sigma_{a,CM} = 1 - \frac{\rho}{1-\rho}\sigma_{b,CM}$ this reduces to

$$\left(1-q\right)\left(r\left(1-\rho\right) - \left(1-r\right)\rho\sigma_{b,HPM} - \left(2r-1\right)\rho\sigma_{b,CM}\right)$$

The difference in success probabilities in state β is

$$\left(1-\gamma_{\beta,CM}\right) - \left(1-\gamma_{\beta,HPM}\right) = \left(1-q\right)\left(-\left(1-r\right)\left(1-\rho\right) + r\rho\sigma_{b,HPM} - \left(2r-1\right)\rho\sigma_{b,CM}\right)$$

Differencing the two differences:

$$\gamma_{\alpha,CM} - \gamma_{\alpha,HPM} - \left(\left(1-\gamma_{\beta,CM}\right) - \left(1-\gamma_{\beta,HPM}\right)\right) = \left(1-q\right)\left(\left(1-\rho\right) - \rho\sigma_{b,HPM}\right) < 0$$

because $\sigma_{b,HPM} > \frac{1-\rho}{\rho}$.

This means that, if the state α success probability decreases when we move from CM to HPM, i.e. $\gamma_{\alpha,CM} - \gamma_{\alpha,HPM} > 0$, then the state β success probability decreases even more. In that case, obviously, $S_{CM} > S_{HPM}$. Hence, assume that the state α success probability increases when we move from CM to HPM, i.e. $\gamma_{\alpha,CM} - \gamma_{\alpha,HPM} < 0$.

In that case, we have

$$\gamma_{\alpha,CM} - \gamma_{\alpha,HPM} < 0$$

which becomes

$$r\left(1-\rho\right) - \left(1-r\right)\rho\sigma_{b,HPM} < \left(2r-1\right)\rho\sigma_{b,CM}$$

Because $\sigma_{b,HPM} > \frac{1-\rho}{\rho}$, this implies

$$\sigma_{b,CM} > \frac{1-\rho}{\rho}$$

But this contradicts the fact that, in all completely-mixed equilibria, $\sigma_{b,CM} < \frac{1-\rho}{\rho}$. Hence, $S_{CM} > S_{HPM}$.

This completes the proof.

Proof of Proposition 11:

1. Welfare ranking in case of multiplicity of equilibria for given n.

Comparing HPM to E: The only difference between HPM and E equilibria is that, in HPM, conflicted voters with a b signal mix, while they always vote expressively in E. Consider one such conflicted voter, i. As i mixes in HPM, the expected payoff from voting instrumentally and expressively must be the same for him. Hence, for purposes of comparing payoffs, we may assume that, in fact, he chooses to vote expressively. Now consider what happens to his payoffs if all the other conflicted voters suddenly started voting expressively instead of mixing. Voter i already receives full expressive payoffs and this does not change. However, using arguments identical to those in the proof of Proposition **??**, it is easily verified that the probability S of electing the better candidate falls. Hence, voter i's payoff also falls. Note, however, that this lower payoff exactly corresponds to i's payoff in E. Finally, all non-conflicted voters' payoffs also fall when S falls. Hence, $W_{HPM} > W_E$.

Comparing LPM to HPM: LPM and HPM equilibria only differ in that conflicted voters with a b signal are more likely to vote expressively in HPM than in LPM. Take one such conflicted voter, i. As i mixes in LPM, the expected payoff from voting instrumentally and expressively must be the same for him. Hence, for purposes of comparing payoffs, we may assume that, in fact, he chooses to vote expressively. Now consider what happens to his payoffs if all the other conflicted voters suddenly started mixing according to $\sigma^A_{b,HPM}$ instead of $\sigma^A_{b,LPM}$. Voter i already receives full expressive payoffs and this does not change. However, using arguments identical to those in the proof of Proposition **??**, it is easily verified that the probability S of electing the better candidate falls. Hence, voter i's payoff also falls. Note, however, that this lower payoff exactly corresponds to i's payoff in HPM. Finally, all non-conflicted voters' payoffs also fall when S falls. Hence, $W_{LPM} > W_{HPM}$.

Comparing CM to HPM: CM and HPM equilibria differ in that conflicted voters with a b signal are more likely to vote expressively in HPM than in CM, while conflicted voters with a a signal always vote expressively in HPM but mix in CM. Consider an individual voter, i. As i mixes in CM whenever he is conflicted, in those cases, the expected payoff from voting instrumentally and expressively must be the same for him. Hence, for purposes of comparing payoffs, we may assume that, in fact, he chooses to vote expressively whenever he is conflicted. Now consider what happens to his payoffs if all the other conflicted voters with a b signal suddenly started mixing according to $\sigma^A_{b,HPM}$ instead of $\sigma^A_{b,CM}$ and all (other) conflicted voters with an a signal suddenly started voting expressively instead of mixing according to $\sigma^A_{a,CM}$. Voter i already receives full expressive payoffs and this does not change. However, using arguments identical to those in the proof of Proposition **??**, it is easily verified that the probability S of electing the better candidate falls. Hence, voter i's payoff also falls. Note, however, that this lower payoff exactly corresponds to i's payoff in HPM. Finally, all non-conflicted voters' payoffs also fall when S falls. Hence, $W_{CM} > W_{HPM}$.

Comparing I to HPM: I and HPM equilibria differ in that conflicted voters always vote instrumentally in I, while, in HPM, conflicted voters with a b signal mix and conflicted voters with an a signal always vote expressively. Consider an individual voter, i. As, conditional on receiving a b signal, i mixes in HPM, his expected payoff from voting instrumentally and expressively must be the same. Hence, for purposes of comparing payoffs, we may assume that, in fact, he chooses to vote expressively. Now consider what happens to his payoffs if all the other conflicted voters suddenly started voting instrumentally. Voter i already receives full expressive payoffs and this does not change. However, using arguments identical to those in the proof of Proposition **??**, it is easily verified that the probability S of electing the better candidate rises. Hence, voter i's payoff also rise. Moreover, because I is an equilibrium, it must be that i's payoff rises even further if he deviated and also started voting sincerely. Finally, all non-conflicted voters' payoffs also rises when S rises. Hence, $W_I > W_{HPM}$.

2. Increasing n within an equilibrium class

As long as we stay within an equilibrium class, the voting behavior of existing voters remains essentially the same when we add additional voters to the electorate, apart from possible changes in the exact equilibrium mixtures probabilities. As before, for purposes of comparing payoffs, we may assume that voters who mix chooses to vote expressively. In that case, the expressive payoffs of existing voters remain unaffected and, therefore, overall welfare only depends on the effect that additional voters have on S. This proves the result.

Moving from one equilibrium class to the next

When there is uniqueness of equilibrium for every n and we ignore integer constraints, then $\{\sigma_a, \sigma_b\}$ move continuously from I to CM to PM to E as n increases. As there are no jumps at the boundaries, the result follows immediately from 2.

When there is multiplicity of equilibrium for some n, but the best equilibrium is selected, we move from I to CM to LPM to E. The only discontinuous jump takes place at $m = \bar{m}_{PM}$, when we move from a LPM equilibrium with $\sigma_b = \sigma_{b,\bar{m}_{PM}}$ to an expressive equilibrium with $\sigma_b = 1$. Clearly, the probability of choosing the better candidate falls at this point. Hence, it remains to show that welfare falls as well. As at $m = \bar{m}_{PM}$, LPM and HPM coincide, the argument is essentially identical to that under 1. "Comparing HPM to E" above.

This completes the proof.

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