Information and Decisions in Social Networks

ESNIE
Cargèse

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Alex Teytelboym

INET, University of Oxford

Do we need networks?

Networks are all around us:

- Financial: banks and financial institutions borrow and lend to each other
- Trading: supply chains can get disrupted
- Physical: infrastructure in cities
- Social: spread of information, disease, good and bad behaviour.

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Networks in institutional economics

Social networks are crucial to the analysis of institutions:

- Social networks and knowledge about investment opportunities (Buchardi and Hassan, QJE 2013)
- Infrastructure, local state capacity and political economy (Acemoglu et al. NBER 2014)
- Roommates and educational outcomes (Sacerdote, QJE 2001)
- Co-author networks and optimal organisational design (Baccara et al., AER 2012)

Buchardi and Hassan, QJE 2013

Acemoglu et al. NBER 2014

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Social networks

"as economists endeavor to build better models of human behavior, they cannot ignore that humans are fundamentally a social species with interaction patterns that shape their behaviors. People's opinions, which products they buy, whether they invest in education, become criminals, and so forth, are all influenced by friends and acquaintances." (M. Jackson, JEP, 2014)

Economic questions about networks

- How does the network form?
  - Static or dynamic network formation
  - Steady state, stability, long-run equilibrium
  - Explain macro-properties from micro-foundations
- How do agents act in the network?
  - Strategic behaviour
  - Bayesian or myopic behaviour
- Holy grail: how do the agents behave when they can change the network?

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Dynamic formation of social networks

Empirics

Most social networks exhibit a variety of macroscopic phenomena:

- Fat-tail degree distribution
  - Loads of popular people.
- High clustering
  - My friends are friends among themselves.
- Assortativity
  - Popular people are friends with popular people
- Low diameter
  - At most "6 degrees of separation"
- Low average shortest-path length
  - On average few "degrees of separation"

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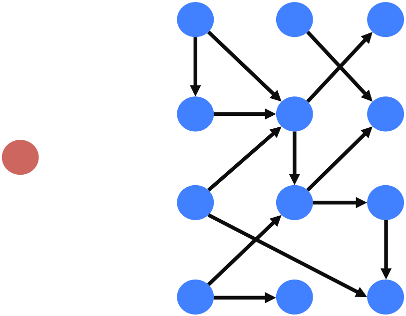
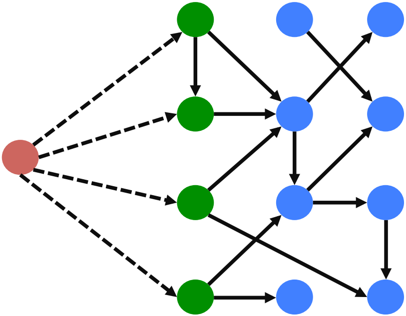
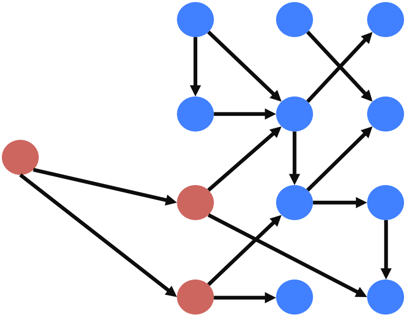
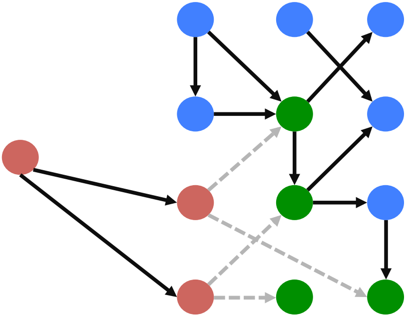
Model

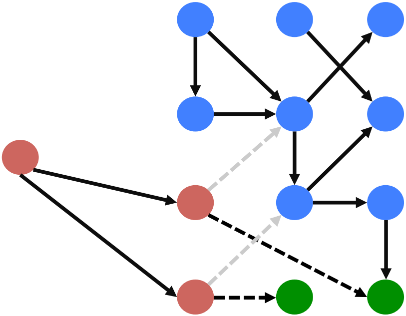
- Start with a bunch of $m_r + m_n + 1$ agents (nodes).
- One agent arrives at time period $t \in \{1, 2, 3...\}$
- Randomly picks m_r friends and links to them with probability p_r
- Randomly picks m_n friends-of-friends and links to them with probability p_n

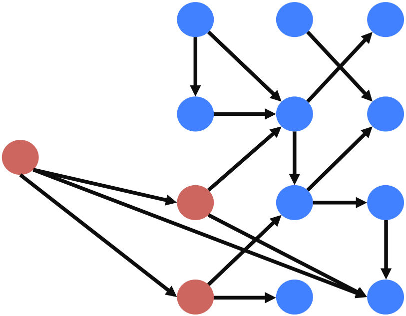












Model

- What is the probability that the node gets an extra in-link at time t (every node has $m_r + m_n$ outlinks)?
- It's approximately:

$$\underbrace{\frac{p_r m_r}{t}}_{i \text{ is picked at random}} + p_n \underbrace{\underbrace{\frac{m_r deg_i(t)}{t}}_{i\text{'s friend } j \text{ picked}} \times \underbrace{\frac{m_n}{m_r(p_r m_r + p_n m_n)}}_{i \text{ picked} \mid j \text{ picked}}}_{i \text{ picked via neighbours}}$$

- We can re-write this as:

$$\underbrace{\frac{p_r m_r}{t}}_{i \text{ is picked at random}} + \underbrace{\frac{p_n m_n}{t} \times \frac{deg_i(t)}{m}}_{i \text{ picked via neighbours}}$$

where $m = p_r m_r + p_n m_n$ is the expected degree ($\equiv$number of in-links).

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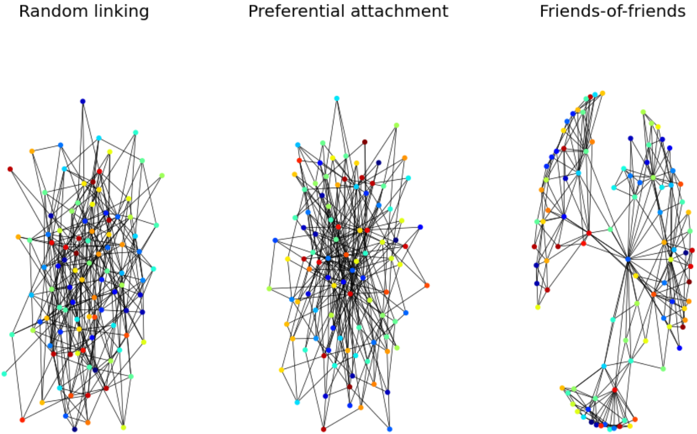
Illustration

Let's fix $m = 6$ and simulate this model for 100 periods.

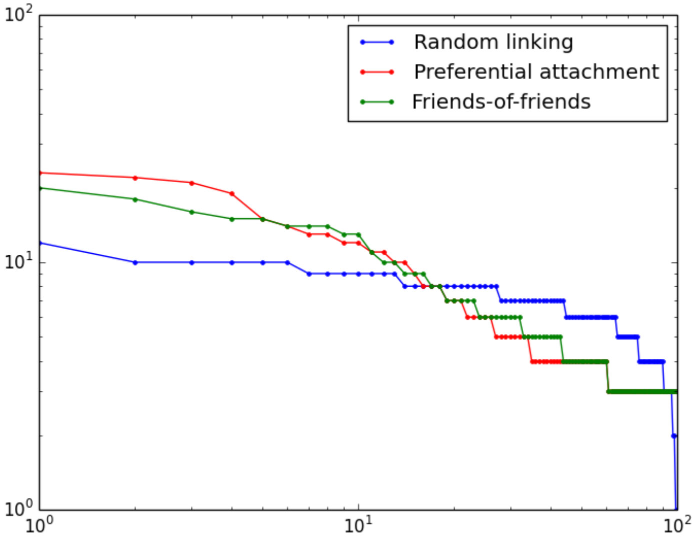
- Case 1 ($p_r = 1$, $p_n = 0$, $r \to \infty$, Random linking): $m = m_r$.
- Case 2 ($p_r = 0$, $p_n = 1$, $r \to 0$, Preferential attachment): $m = m_n$
- Case 3 ($p_r = p_n = 1$, $r = 1$, Friends of friends): $m = m_r + m_n$

where $r = \frac{p_r m_r}{p_n m_n}$ is the ratio of random links to links via friends-of-friends.

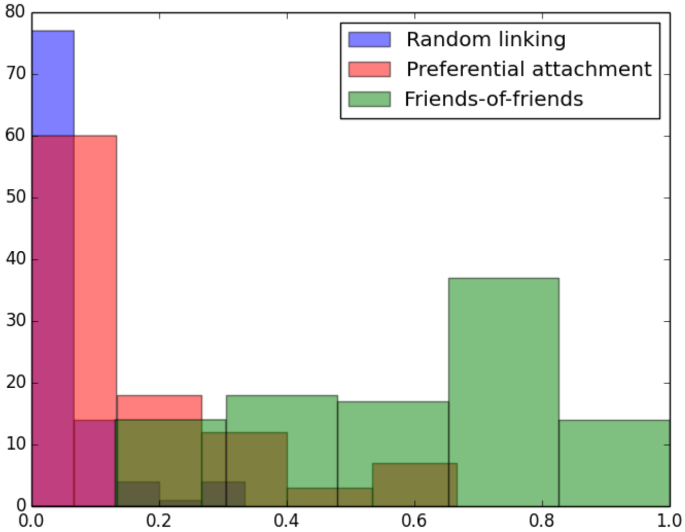
Graphs


Random linking
Preferential attachment
Friends-of-friends

Degree distribution



Clustering



80
70
60
50
40
30
20
10
0
0.0
0.2
0.4
0.6
0.8
1.0
Random linking
Preferential attachment
Friends-of-friends

Solving the model

- In order to solve this model, we use a **mean-field approximation**.
- Assume that agent i's degree evolves according to the following diffential equation...

$$\frac{ddeg_i(t)}{dt} = \frac{p_r m_r}{t} + \frac{p_n m_n}{t} \times \frac{deg_i(t)}{m}$$

- ...which solves to give the cumulative distribution of in-links at any time period t:

$$F_t(d) = 1 - \left(\frac{d_0 + rm}{d + rm}\right)^{1+r}$$

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Calibrating the model

- Empirically calculate m and fix it.
- Then calibrate r to fit the equation for F to the empirical cumulative degree distribution.
- Then, holding the calibrated r fixed, calibrate $p = p_r = p_n$ to fit the clustering coefficient (p doesn't affect r and the degree distribution, but affects clustering).
- If the model is good, then the simulated average shortest-path length and diameter should match the data.

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Results for 6 social networks

TABLE 1—PARAMETER ESTIMATES ACROSS APPLICATIONS

Dataset:	WWW	Citations	Coauthor	Ham radio	Prison	High-school romance
Number of nodes:	325729	396	81217	44	67	572
Avg. in-degree: m	4.6	5.0	0.84	3.5	2.7	0.83
r from Fit	0.57	0.63	4.7	5.0	∞	∞
p from Fit	0.36	0.27	0.10	1	1	—
R^2 of Fit	0.97	0.98	0.99	0.94	0.94	0.99
Avg. clustering data	0.11	0.07	0.16	0.47	0.31	—
Avg. clustering fit	0.11	0.07	0.16	0.22	0.10	—
Diameter data	11.3 (avg)	4	26	5	7	—
Diameter fit	(6, 12)	(4, 8)	(19, 38)	(4, 8)	(5, 10)	(12, 24)

Where is the economics?

Proposition

Assume that agents' utility is increasing and concave in their in-degree.
With m fixed, society welfare is higher when r is higher.
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Further steps

- Homophily: my friends are similar to me.
  - Race, gender, income, employment, political opinions.
  - "Propinquity" vs. "Preference"
- Friending (Tarbush and Teytelboym, 2014): model of online social networks
- Agents make friends within social groups: can calibrate the parameters of "how long a student makes friends in her class vs. her dorm".
- Derive the **dynamics** of homophily.
- Test using Facebook data, v. good fit.

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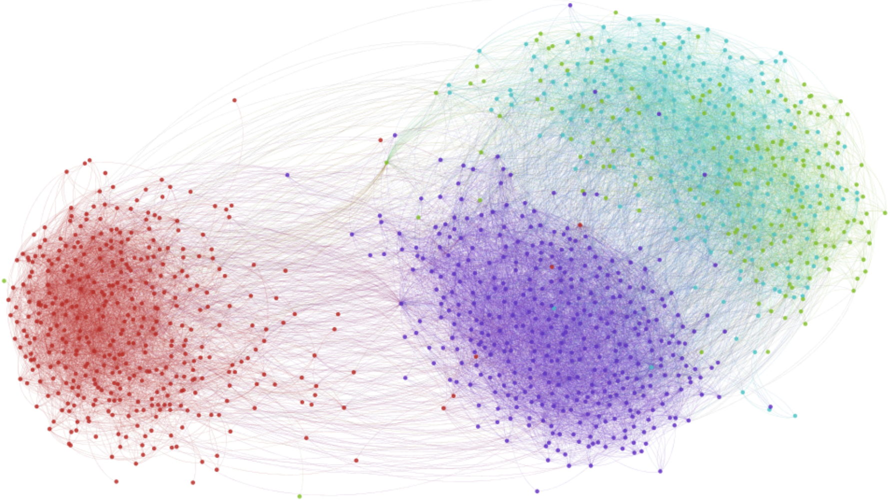
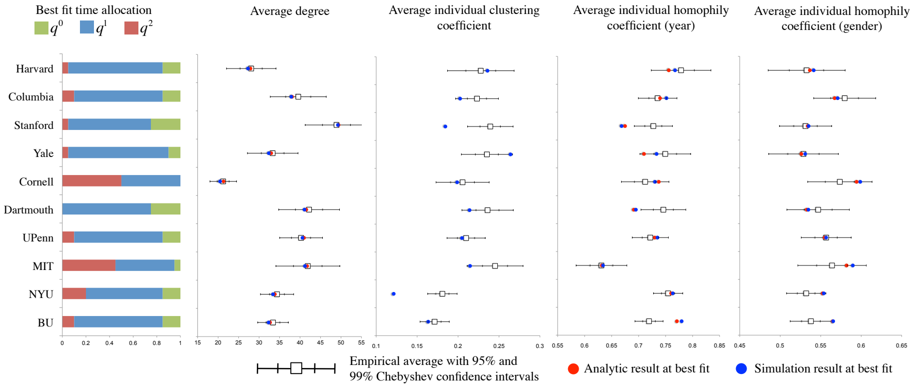
Baseline empirical observations: Harvard



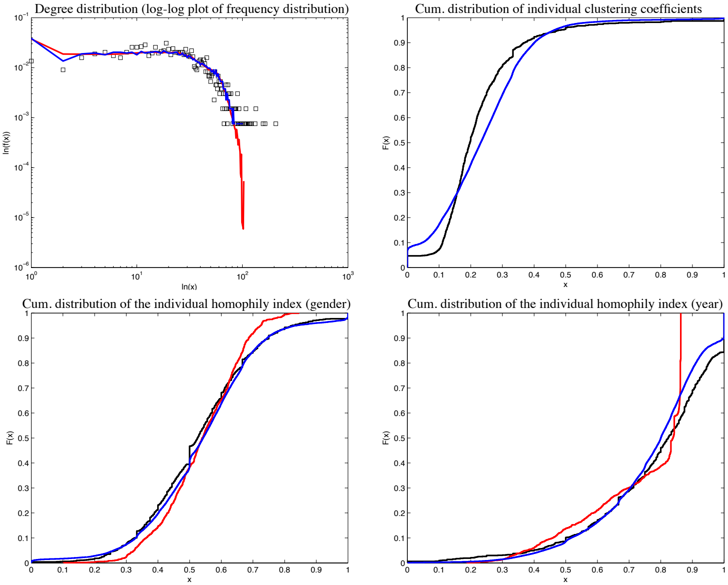
Year of graduation: Red - 2009; Purple - 2008; Blue - 2007; Green - 2006

All colleges



The individual clustering coefficient for i is the proportion of agent i's friends, who are friends with each other.

Harvard



Black: empirical Red: analytical Blue: simulation

Behaviour in fixed networks

Motivation

- Agents plays a game on a **fixed** network.
- My neighbors characteristics and their actions affect my payoffs.
- Examples:
  - Public good provision
  - Criminal behaviour
  - Choice of fashion
  - Platform adoption
- But my behaviour affects your behaviour and your behaviour affects mine: is everything endogenous? This is known as the "reflection problem" (Manski, 1993).

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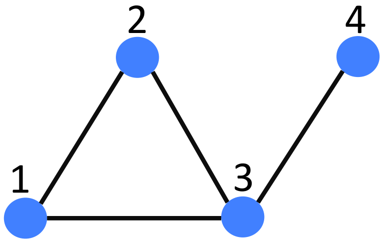
Notation

- N agents
- $y_i \in \mathbb{R}$ is i's action
- $x_i \in \mathbb{R}$ publically observed characteristic
- $\epsilon_i \in \mathbb{R}$ privately observed characteristic (i's type)

Notation

- **A** is a matrix of peer and contextual effects.
- Matrix represents a undirected, unweighted graph.
- Why is a network a matrix? $a_{ij} > 0$ if i is influenced by j.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$


2
4
1
3

Notation

- Let's "row-normalise" **A**

$$\mathbf{G} = \begin{bmatrix} \mathbf{g_1} \\ \mathbf{g_2} \\ \mathbf{g_3} \\ \mathbf{g_4} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Complementarities: social multiplier

$$U_i(y_i, y_{-i}) = \left(\alpha + \beta x_i + \epsilon_i + \underbrace{\delta \sum_j g_{ij} x_j}_{\text{contextual effect}} \right) y_i - \underbrace{\frac{1}{2} y_i^2}_{\text{cost of action}}$$

$$+ \underbrace{\phi \sum_j g_{ij} y_i y_j}_{\text{strategic complements}}$$

Solve the models

- For unique (Bayes) Nash equilibrium: $0 < \phi < 1$
  FOCs:

$$
\begin{aligned}
0 &= \alpha + \beta x_i + \epsilon_i + \delta \mathbf{g}_i \mathbf{x} + \phi \mathbf{g}_i \mathbf{y}^* - y_i^* \\
\mathbf{y}^* &= [\alpha \mathbf{1} + \beta \mathbf{x} + \delta \mathbf{G} \mathbf{x} + \phi \mathbf{G} \mathbf{y}^* + \boldsymbol{\epsilon}] \\
\mathbf{y}^* &= (\mathbf{I} - \phi \mathbf{G})^{-1} [\alpha \mathbf{1} + \beta \mathbf{x} + \delta \mathbf{G} \mathbf{x} + \boldsymbol{\epsilon}]
\end{aligned}
$$

Identification

- Let's focus on the social multiplier model. The reduced-form is:

$$\mathbf{y} = (\mathbf{I} - \phi\mathbf{G})^{-1}\left[\alpha\mathbf{1} + \beta\mathbf{x} + \delta\mathbf{G}\mathbf{x} + \boldsymbol{\epsilon}\right]$$

- Assume $\mathbb{E}(\boldsymbol{\epsilon}|\mathbf{x}) = 0$, outer product $\mathbf{1}\mathbf{x}^{\mathsf{T}}$ has full rank, don't assume homoskedasticity.
- Can we identify $\boldsymbol{\theta} = (\alpha, \beta, \delta, \phi)$?

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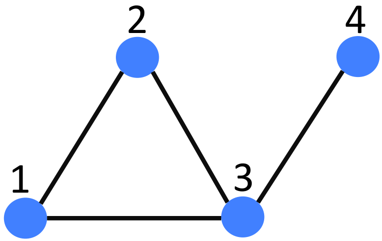
Identification

- Good news!

Proposition (Bramoullé et al. (2009))

Suppose that $\beta\phi + \delta \neq 0$. If $\mathbf{I}, \mathbf{G}, \mathbf{G}^2$ are linearly independent, then the model parameters are identified.

- $\mathbf{G^2x}$ - characteristics of friends-of-friends who're not my friends - can be an instrument for $\mathbf{Gy}$.


2
4
1
3

Identification

- Why? Let's expand:

$$\mathbf{y} = (\mathbf{I} - \phi\mathbf{G})^{-1}[\alpha\mathbf{1} + \beta\mathbf{x} + \delta\mathbf{G}\mathbf{x} + \boldsymbol{\epsilon}]$$

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- Blume et al. (2015) show that, in fact, whenever $\mathbf{G}$ is known, identification is **generic**.

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Identification under other conditions

We can identify the model under a host of other conditions:

- Peer effects and contextual effects matrices are different (Blume et al., 2015).
- **G** is not row-normalised: local aggregate rather than local average effects (Boucher, 2014; Liu et al, 2015).
  - Can even have local aggregate effects and conformity in one model!
- Correlated effects (Bramoullé et al., 2009): α_l is network specific.

Empirical example: school kids

Classic paper by Bramoullé et al. (JoE 2009) using Add Health data:

- 132 middle and high schools in the US. We will work within each school.
- Median school size is 240 students; 55,000 links in total.
- Rich socio-economic and demographic data on students and their parents.
- List up to 5 male and 5 female friends.

Empirical example: school kids

- Estimate effect of my friends on my "recreational activities" (sports clubs, societies etc.)

$$\mathbf{y} = \mathbf{X}\beta + \mathbf{G}\mathbf{X}\delta + \phi\mathbf{G}\mathbf{y} + \epsilon$$

- Aside: in the paper, because there are school network fixed effects, authors apply a fixed-effect transformation:

$$(\mathbf{I} - \mathbf{G})\,\mathbf{y} = \phi\,(\mathbf{I} - \mathbf{G})\,\mathbf{G}\mathbf{y} + (\mathbf{I} - \mathbf{G})\,\mathbf{X}\beta + (\mathbf{I} - \mathbf{G})\,\mathbf{G}\mathbf{X}\delta + \boldsymbol{\nu}$$

  and technically stricter conditions are needed for identification. But let's not worry!

Empirical example: school kids

- Instruments for the endogenous variable **Gy** are

$$\mathbf{Z} = \left[\mathbf{X}, \mathbf{GX}, \mathbf{G}^2\mathbf{X}\right]$$

- Instrumented variables are

$$\tilde{\mathbf{X}} = [\mathbf{X}, \mathbf{GX}, \mathbf{Gy}]$$

- Obtain using standard 2-stage least squares:

$$\hat{\theta}^{2SLS} = (\tilde{\mathbf{X}}'\mathbf{P_Z}\tilde{\mathbf{X}})^{-1}\tilde{\mathbf{X}}\mathbf{P_Z}\mathbf{y}$$

where $\mathbf{P_Z} = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}$ is the projection matrix.

- The model is over-identified, so you need to do a little more to fix it. Again, let's ignore that!

Empirical example: school kids

Highlights of the results:

- A student's recreational activities index decreases with the mean age of his friends but rises with their mean parents' participation in the labor market.
- 1 point increase in friends activity $\Rightarrow$ 0.446 increase in my activity (and significant).
- Quality of estimation is greatly affected by network properties (density is bad for precision; clustering is ambiguous)!

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Key players

- We can identify key players - those that should be targeted in an intervention by knowing the Nash equilibrium of the game.
- Consider a general fomulation of both of our models:

$$\mathbf{y} = \tau(\mathbf{I} - \phi\mathbf{G})^{-1}(\alpha\mathbf{1} + \mathbf{X}\beta + \epsilon)$$

- Let's introduce a shock ς. The impact of the shock on any agent i:

$$\tau\varsigma\mathbf{1}'\left(\mathbf{I} - \phi\mathbf{G}\right)^{-1}\mathbf{e_i}$$

- The total shock is proportional to the sum of the entries of this vector:

$$\mathbf{b} = \left(\mathbf{I} - \phi\mathbf{G}\right)^{-1}\mathbf{1}$$

- This vector **b** is Page-Rank, which Google uses to measure how important web-pages are.
- Policy: estimate ϕ and target individuals with highest Page-Rank!

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Conformity

How about another model?

$$U_i(y_i, y_{-i}) - = \left(\alpha + \beta x_i + \epsilon_i + \underbrace{\delta \sum_j g_{ij} x_j}_{\text{contextual effect}} \right) y_i - \underbrace{\frac{1}{2} y_i^2}_{\text{cost of action}}$$

$$- \frac{\lambda}{2} \left(\underbrace{y_i - \sum_j g_{ij} y_j}_{\text{conformity}} \right)^2$$

Assumptions

- For unique (Bayes) Nash equilibrium to exist: $\lambda \geq 0$
  FOCs:

$$
\begin{aligned}
0 &= \alpha + \beta x_i + \epsilon_i + \delta \mathbf{g}_i \mathbf{x} - \lambda \left(y_i^* - \mathbf{g}_i \mathbf{y}^* \right) - y_i^* \\
y_i^* &= \frac{1}{1+\lambda} \left[\alpha + \beta x_i + \epsilon_i + \delta \mathbf{g}_i \mathbf{x} + \lambda \mathbf{g}_i \mathbf{y}^* \right] \\
\mathbf{y}^* &= \left(\frac{1}{1+\lambda} \right) \left[\alpha \mathbf{1} + \beta \mathbf{x} + \delta \mathbf{G} \mathbf{x} + \lambda \mathbf{G} \mathbf{y}^* + \boldsymbol{\epsilon} \right] \\
\mathbf{y}^* &= \left(\mathbf{I} - \frac{\lambda}{1+\lambda} \mathbf{G} \right)^{-1} \left(\frac{1}{1+\lambda} \right) \left[\alpha \mathbf{1} + \beta \mathbf{x} + \delta \mathbf{G} \mathbf{x} + \boldsymbol{\epsilon} \right]
\end{aligned}
$$

Oops

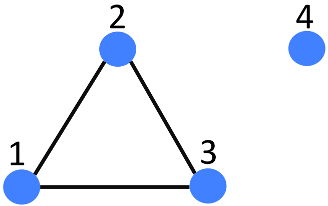
- Hold on...

$$\mathbf{y}^* = \left(\mathbf{I} - \frac{\lambda}{1+\lambda}\mathbf{G}\right)^{-1} \left(\frac{1}{1+\lambda}\right) [\alpha\mathbf{1} + \beta\mathbf{x} + \delta\mathbf{G}\mathbf{x} + \boldsymbol{\epsilon}]$$

But if you remember, the social multiplier model is:

$$\mathbf{y}^* = (\mathbf{I} - \phi\mathbf{G})^{-1} [\alpha\mathbf{1} + \beta\mathbf{x} + \delta\mathbf{G}\mathbf{x} + \boldsymbol{\epsilon}]$$

Models are observationally identical! But completely different policy implication!
How can we identify the social multiplier ϕ or the conformity parameter λ?
You can **use OLS on isolated** individuals!


2
4
1
3

Endogenous networks

- This gets hard very quickly, but just to give you a flavour...
- Is the network even endogenous?
- Suppose there is an unobserved characteristic of agents η that affects their behaviour **and** the probability of a link.

$$y = \mathbf{X}\boldsymbol{\beta} + \mathbf{GX}\boldsymbol{\delta} + \rho\boldsymbol{\eta} + \boldsymbol{\xi}$$

- Pretend link the network is exogenous
  - Predict the links $\hat{a}_{ij}$ using the observed characteristics - look at incorrect predictions.
  - Predict the outcomes using the observed characteristics - look at residuals $\hat{\epsilon}_{ij}$.
- Roughly speaking, if $\hat{a}_{ij}$ and $\hat{\epsilon}_{ij}$ are correlated, this indicates homophily and endogenous links.

Social network data

- SNAP - Stanford Network Analysis Project: loads and loads of social network datasets to play with. snap.stanford.edu/
- Add Health: over 100 schools, very detailed data ($<$10 friends) http://www.cpc.unc.edu/projects/addhealth/data
- Detailed social network data for 75 Indian villagese ($\approx$20-30 friends) http://economics.mit.edu/faculty/eduflo/social
- Twitter/Digg/Flickr http://www.isi.edu/integration/people/lerman/downloads.html
- Facebook: can still find it (first 100 colleges, complete within-college networks, very anonymized).
- Collect it yourself
  - Scrape it (quick and dirty, learn a bit of Python and Javascript)
  - Surveys (patience and money)
  - Contact companies and ask for it (charm and added valued)

References I