Collective choices on the market

The interplay between voting and trading

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A puzzle

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- Reality is hard to reconcile with prediction of social choice theory, spotted with impossibility results.

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Collective	2746	14.8	100	94.8	87.1
Individual	268	53.2	99.9	88.8	78
Nomination	818	9.2	100	83.7	86.4
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- One possible answer: Markets are perfect and complete.

General equilibrium analysis

Alignment and conflict on the market

The benchmark model (Arrow-Debreu)

- Economy with ℓ goods: $s \in \{1, \ldots, \ell\}$.
- J firms $\mathcal{J} = \{1, \ldots, J\}$; production sets $Y_j \subset \mathbb{R}^\ell$;
  - a list of production plans: $y = (y_j)_{j \in \mathcal{J}} \in \mathbb{R}^{\ell J}$.
- I consumers $\mathcal{I} = \{1, \ldots, I\}$; utility functions $U_i : \mathbb{R}^\ell \to \mathbb{R}$;
  - U_i differentiable, increasing and strictly quasi-concave.
  - endowment: $\bar{x}_i \in \mathbb{R}^\ell$;
  - initial portfolio of shares: $\bar{\theta}_i \in \mathbb{R}^J$;
  - a list of consumption bundles: $x = (x_i)_{i \in \mathcal{I}} \in \mathbb{R}^{\ell I}$;
  - a list of portfolios: $\theta = (\theta_i)_{i \in \mathcal{I}} \in \mathbb{R}^{IJ}$.

Allocations

Definition

An **allocation** is a list of consumption bundles and production plans (x, y) such that aggregate demand and supply match:

$$\sum_i x_i = \sum_i \bar{x}_i + \sum_j y_j.$$

Definition

An allocation (x, y) is **Pareto optimal** if there is no other allocation (x', y') such that $U_i(x_i') \geq U_i(x_i)$ for all i (with at least one strict inequality).

Value vectors

- Agents use **value (price) vectors** $\nabla \in \mathbb{R}^{\ell}_{+} \setminus \{0\}$.
- Sometimes normalized (in the simplex):

$$\mathbb{S}^{\ell} \;=\; \left\{ \nabla \in \mathbb{R}^{\ell}_{+} \mid \sum_s \nabla^s = 1 \right\}.$$

Definition

Production plan $y_j \in Y_j$ is **optimal** for the value vector ∇_j if it maximizes the value of production for ∇_j:

$$y_j = \arg\max \left\{ \nabla_j \cdot y_j' \mid y_j' \in Y_j \right\}.$$

- To fix ideas: the value vector points in the ideal direction toward which the agent would like to change the production.

Modelling production

- **Smooth production**: Y_j is non-empty and compact; and there is a smooth and strictly convex map $g_j : \mathbb{R}^\ell \to \mathbb{R}$ such that:

$$Y_j = \{ y_j \in \mathbb{R}^\ell \mid g_j(y_j) \leq 0 \}.$$

Lemma

There is a one-to-one mapping between value vectors in $\mathbb{S}^\ell$ and optimal plans in ∂Y_j.

Equilibrium

- $A = (y_1 \dots y_J)$ the $\ell \times J$ matrix of plans; the initial resources of i :

$$\bar{x}_i + A\bar{\theta}_i.$$

- For market prices $p \in \mathbb{S}_+^\ell$, budget set of i :

$$P_i(p, y) = \left\{ x_i \in \mathbb{R}^\ell \mid p \cdot x_i \;\leq\; p \cdot (\bar{x}_i + A\bar{\theta}_i) \right\}.$$

Definition

An **equilibrium** (for fixed plans $\bar{y}$) is a price and an allocation $(p^*, x^*, \bar{y})$ such that individual consumers optimize: for all i

$$x_i^* = \arg\max\{\, U_i(x_i) \mid x_i \in P_i(p^*, \bar{y})\}.$$

- First and second welfare theorems hold.

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- But this is a static model, what if uncertainty?

Arrow-Debreu continued

- Economy with two dates $t \in \{0, 1\}$ and uncertainty at date 1, with ℓ states of the world $\{1, \ldots, \ell\}$;
  - no consumption or production at date 0;
  - one commodity in every state at date 1;
  - 'good s' is now the commodity in state s;
- consumers *trade shares* (and not commodities) given *asset prices*.

Financial equilibrium

- Given asset prices q and plans y, budget set of consumer i is:

$$Q_i(q,y) = \left\{ x_i \in \mathbb{R}^\ell \mid \exists \theta_i \in \mathbb{R}^J \text{ s.t. } q \cdot \theta_i \leq q \cdot \bar{\theta}_i \text{ and } x_i = \bar{x}_i + A\theta_i \right\}.$$

Definition

An equilibrium (for fixed plans $\bar{y}$) is an asset price vector and an allocation $(q^*, x^*, \bar{y})$ such that individual consumers optimize: for all i

$$x_i^* = \arg\max\{ U_i(x_i) \mid x_i \in Q_i(q^*, \bar{y})\}$$

Theorem

At equilibrium: for all i

$$x_i^* - \bar{x}_i \in \langle A \rangle \text{ and } DU_i(x_i^*) \in \left\langle \left(A^T\right)^{-1} (q^*) \right\rangle.$$

Complete market

- Markets are complete when A has rank ℓ (hence necessarily $J \geq \ell$).
- Then all shareholders are aligned: for all i

$$\nabla_{\|i}(x_i^*) = \nabla^*$$

where ∇^* is the *unique* no arbitrage value vector in $\mathbb{S}_+^\ell$.

Definition

A general (financial) equilibrium (for complete market) is an equilibrium (q^*, x^*, y^*) such that for all j,

$$y_j^* = \arg\max \left\{ \nabla^* \cdot y_j \mid y_j \in Y_j \right\}.$$

Incomplete market

Theorem

Fix $\bar{y}$. Assume there are at least 2 consumers ($I \geq 2$). Then for almost all $\bar{x}$, the present value vectors of consumers are distinct two by two:

$$\nabla_{\|^i}(x_i^*) \neq \nabla_{\|^{i'}}(x_{i'}^*) \text{ if } i \neq i'$$

at every equilibrium $(q^*, x^*, \bar{y})$.

- Shareholders use different vectors to compute values.

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- Shareholders use different vectors to compute values.
- *No more alignement of shareholders.*
- How should production plans be chosen?

And what if imperfect competition: some firms have market power?

- Income effect *and price effect:*

$$\boxed{\nabla_i(p^*, x_i^*, y) \;= \bar{\theta}_i p^* + \; D_y \breve{p}^T(\bar{x}_i + \bar{\theta}_i y - x_i^*)}$$

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- Unified framework to study many types of market failures at once:

the value vectors $((\nabla_i)_{i \in \mathcal{I}}, (\nabla_j)_{j \in \mathcal{J}})$.

In case of market failures

- We are left with a *collective choice problem* within each firm j :
  - choice set: Y_j
  - voters: shareholders, stakeholders, etc.? $(\mathcal{I}_j)$
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  - (voting) weight in the decision process: one share-one vote, one person-one vote, etc. ? $(\lambda_{ij})_{i \in \mathcal{I}_j}$

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  - Can the collective choice be efficient?

Social choice analysis

Democracy in the firm

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- In the spirit of the equilibrium analysis, a central question is: do we have *stable* plans?
- Focus: (super) majority voting with rate $\rho \in [0.5, 1]$

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Definition

Given value vectors $((\nabla_i)_{i \in \mathcal{I}}, (\nabla_j)_{j \in \mathcal{J}})$, and a rate $\rho \in [0.5, 1]$:
- ∇_j respects the $-$**majority principle** if $\forall \Delta y \in \mathbb{R}^\ell$, $\lambda_j(\mathcal{I}(\Delta y)) > \rho$ implies $\nabla_j \cdot \Delta y > 0$.
- $(\nabla_j)_{j \in \mathcal{J}}$ is $\rho-$**majority stable** if $\forall j$ ∇_j respects the $\rho-$majority principle.

$\rho-$majority equilibrium

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A $\rho-$**majority equilibrium** is an equilibrium (p^*, x^*, y^*) such that the value vectors $(\nabla_j^*)_{j\in\mathcal{J}}$ supporting y^* are $\rho-$majority stable.

- Equilibrium concept based on
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  - we characterize (politically) stable decisions;
  - the only ones likely to be regularities.

Optimal rate of super majority

- If ρ is too high, too many equilbria: too conservative.
- If ρ is too low, no equilibrium: chaos.

- What is the optimal rate for corporate charter?

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- If ρ is too low, no equilibrium: chaos.
- What is the optimal rate for corporate charter?
- Philosophy: the optimal ρ^* is the lowest that guarantees existence of an equilibrium.
- The question: 'Does trading help the collective choice?' translates in:

By how much does trading reduce ρ^*?

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$$\rho^* = (\ell - 1)/\ell$$

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- $\rho^* = 0.5$ if distribution of $(\nabla_i)_{i \in \mathcal{I}}$ is:
  - unidimensional (median voter theorem, $\ell = 1$);

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- Caplin & Nalebuff (1988, 1991) – multivariate *mean* voter theorem:

  If distribution of $(\nabla_i)_{i\in\mathcal{I}}$ is $\sigma -$ concave then $\rho^* = 0.64$.

- $\rho^* = 0.5$ if distribution of $(\nabla_i)_{i\in\mathcal{I}}$ is:
  - unidimensional (median voter theorem, $\ell = 1$);
  - or centrally symmetric.

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- Does trading reduce polarization?

Political economy of the firm

The symbiosis of trading and voting

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Theorem

All economies have a $\rho-$majority stable equilibrium if and only if
$$\rho \ \geq \ \frac{\ell - J}{\ell - J + 1}$$

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Theorem

Suppose that $\mathcal{I}$ is compact and convex, with a σ-concave density. There exists a $0.64-$majority equilibrium.

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Theorem

Under the shareholder governance, if the distribution of $(\nabla_i)_{i \in \mathcal{I}}$ is σ-concave on a compact and convex support, then (quasi) constrained efficient equilibria are $0.64-$majority stable.

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Consider an economy.
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Then for the shareholder governance there exist ρ-majority equilibria with perfect internalization.
Suppose

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Then for the stakeholder democracy there exist ρ-majority equilibria with perfect internalization.

Can voting be efficient? (Imperfect competition)

- Value vectors depend of *net supplies*: $z_i^* = \bar{x}_i + \bar{\theta}_i y - x_i^*$

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Reciprocal aggregation

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- **Reciprocal aggregation**: fixed points along this loop.

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  - Suppose we are at a market equilibrium; consider the change of plan $\Delta y \in \mathbb{R}^\ell$ in a generic firm.
  - If all shareholder prefer the change, then the firm should prefer it too.

The (strong) unanimity rule

Definition

Given value vectors $((\nabla_i)_{i\in\mathcal{I}}, (\nabla_j)_{j\in\mathcal{J}})$:
- ∇_j respects the **Pareto principle** (across shareholders) if $\forall \Delta y \in \mathbb{R}^\ell$, $\nabla_i \cdot \Delta y \geq 0$ for every $i \in \mathcal{I}_j$ (with at least one $>$) implies $\nabla_j \cdot \Delta y > 0$.
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- Define the *positive cone* of a family of vectors $(\nabla_i)_{i\in\mathcal{I}}$

$$K_+\left\{(\nabla_i)_{i\in\mathcal{I}}\right\} = \left\{\nabla = \sum\nolimits_{i\in\mathcal{I}} \mu_i \nabla_i \text{ , with } \mu_i > 0 \text{ for all } i\right\}$$

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- (Firms do not "think out of the box".)

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- When appropriate, they use them to update their own present values. In short: they aggregate them.
- Georg Simmel: "[...] *as individuals, we form the personality out of particular elements of life, each of which has arisen from, or is interwoven with, society. This personality is subjectivity par excellence in the sense that it combines the elements of culture in an individual manner. There is here a reciprocal relation between the subjective and the objective. As the person becomes affiliated with a social group, he surrenders himself to it. A synthesis of such subjective affiliations creates a group in an objective sense. But the person also regains his individuality because his pattern of participation is unique: hence the fact of multiple group-participation creates in turn a new subjective element. Causal determination of, and purposive action by, the individual appear as two sides of the same coin*."

Pareto principle accross firms

- In assessing the opportunity of a change in production $\Delta y \in \mathbb{R}^S$, shareholder i faces $|\mathcal{J}_i|$ potentially different evaluations represented by the present value vectors $(\nabla_j)_{j \in \mathcal{J}_i}$.

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- Individual do not 'think out of the box'.

Stable states and alignment of shareholeders

Definition

A state (U, θ, x, y) is **stable** provided:
- $(\nabla_j(y_j))_{j\in\mathcal{J}}$ is Pareto stable across shareholders;
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Suppose (U, θ, x, y) is stable and $\mathcal{G}(\theta)$ is connected. Then $\exists \nabla$ s.t. $\forall i, j$

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- Markets do not fail...

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For θ, a **cluster** is a nonempty $\mathcal{C} \subset \mathcal{I} \cup \mathcal{J}$ with $\cup_{i \in \mathcal{C}} \mathcal{J}_i \cup \cup_{j \in \mathcal{C}} \mathcal{I}_j = \mathcal{C}$ s.t. $\mathcal{D} \subset \mathcal{C}$ and $\cup_{i \in \mathcal{D}} \mathcal{J}_i \cup \cup_{j \in \mathcal{D}} \mathcal{I}_j = \mathcal{D}$ imply $\mathcal{D} = \mathcal{C}$ or $\mathcal{D} = \emptyset$.

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Corollary

Suppose (U, θ, x, y) is a stable state. Then $\forall\ \mathcal{C}\ \exists \nabla_{\mathcal{C}}$ s.t. $\forall i, j \in \mathcal{C}$

$$\nabla_i(U_i, x_i) = \nabla_j(y_j) = \nabla_{\mathcal{C}}.$$

Stable states and alignment of shareholders

Definition

For θ, a **cluster** is a nonempty $\mathcal{C} \subset \mathcal{I} \cup \mathcal{J}$ with $\cup_{i \in \mathcal{C}} \mathcal{J}_i \cup \cup_{j \in \mathcal{C}} \mathcal{I}_j = \mathcal{C}$ s.t. $\mathcal{D} \subset \mathcal{C}$ and $\cup_{i \in \mathcal{D}} \mathcal{J}_i \cup \cup_{j \in \mathcal{D}} \mathcal{I}_j = \mathcal{D}$ imply $\mathcal{D} = \mathcal{C}$ or $\mathcal{D} = \emptyset$.

Corollary

Suppose (U, θ, x, y) *is a stable state. Then* $\forall\ \mathcal{C}\ \exists \nabla_{\mathcal{C}}$ *s.t.* $\forall i, j \in \mathcal{C}$

$$\nabla_i(U_i, x_i) = \nabla_j(y_j) = \nabla_{\mathcal{C}}.$$

- Two disagreeing shareholders never meet in a GAM.

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  - we had: $(\bar{\theta}, \bar{x}) \longrightarrow (\theta^*, x^*)$; now we need: $(\bar{U}, \bar{\theta}, \bar{x}) \longrightarrow (U^*, \theta^*, x^*)$

The expected uility framework: updating (subjective) beliefs

- Consumers have subjective beliefs $\pi_i \in \triangle$ and utility $u_i : \mathbb{R} \to \mathbb{R}$ accounting for their "tastes"; denote $\vec{u}_i(x_i) = (u_i(x_i^s))_s$

$$U_i(x_i) = \pi_i \cdot \vec{u}_i(x_i) \quad = \sum_{s \in \mathcal{S}} \pi_i^s u_i(x_i^s).$$

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- Then the cone condition reads: $\pi_i \; = \; \sum_{j \in \mathcal{J}_i} \nu_{ij} \pi_j$: mimics the DeGroot (1974) model of beliefs formation on a graph, where shareholder i gives 'weight' ν_{ij} to the belief in firm j.

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