Lobbying in Corsica: A First Pass

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What Interest Groups Do and Why They Do It?

Question 1: **How and why do interest groups (IGs) form?**

Question 2: **What is their impact on public decision-making?**

*Surprisingly...*Those two questions have been treated separately in both the political science and economics literatures.

Question 1: The "Olson Tradition"

→ Olson (1965). *"The Logic of Collective Action: Public Goods and the Theory of Groups."*

- **The problem of collective action: The free-riding problem.**

  → Aggregation of individual preferences may fail even with individuals having congruent interests. Difference between individual and collective rationality.

- **Take-away:** The so-called *Olson's Paradox* → Large unorganized groups may be exploited by small organized ones.

Question 2: "Pluralistic Politics"

→ Dahl (1961). *"Who Governs?"*

- **What do IGs do?**

  → How are the preferences of diverse groups aggregated?

  → Is competition between IGs any good? And why?

- **Take-away:** Competition implies efficient decision-making.

→ Very much a *"Chicago School"*'s view of politics.

Those Two Views Are Partial (1)

- Olson (1965): **Each group is taken in isolation.**
    - Silent about the impact of competition on the problem of collective action.
    - Costs and benefits of formation are of course **endogenous** to the political process.
- Dahl (1961): **Competition between already organized groups.**
    - Silent about the impact of the collective action problem on competition between groups.
    - The representation of interests **may thus be biased**.

The View of Political Scientists

Baumgartner and Leech (1998, pp. 88): *"One sees the interest-group system as hopelessly biased in favor of powerful economic interests and narrow special pleaders; another sees a greater diversity of interest in the Washington policy community and a positive role for groups in the creation of better citizens.* **The relative emphasis that scholars have placed on each of these views has changed from decade to decade, but neither has been shown to be completely accurate: a complete view must recognize elements of both views."**

Economic Literature: *"Olson"* Tradition and the Modern Theory of Free Riding

- **Public good provision under asymmetric information**:

  → Clarke (1971), Groves (1973), Laffont and Green (1977), D'Aspremont and Gérard-Varet (1979), Laffont and Maskin (1982), Mailath and Postlewaite (1990)....

  - **Asymmetric information** on preferences as a rationale for the free-riding problem.

    VETO/PARTICIPATION CONSTRAINT+ BAYESIAN INCENTIVE COMPATIBILITY+ BUDGET BALANCE⇒**Inefficiency...**

- A *"partial" equilibrium approach:* Not really targeted to discuss lobbying groups formation in a broader equilibrium framework.

  → **Endogenous stakes.**

Literature (2): *"Dahl"* Tradition

→ **Common Agency**: Bernheim and Whinston (1986), Grossman and Helpman (1994, 2001), Laussel and Le Breton (1998, 2001).

- **Competition among IGs perfectly aggregates interests.**
- **No free-riding**. Neither **within** groups (*by assumption, formation is not an issue*), nor **between** groups (more subtle).
- Implementation: **"Truthful" Equilibria**. $T_i(q) = \max\{S_i(q) - C_i, 0\}$. $\Rightarrow$ **Efficiency**.

The Common Agency Model

Theory: Bernheim and Whinston (1986), Laussel and Le Breton (1998, 2001).

Applications: International trade (e.g., Grossman and Helpman (1994, 1995, 2001), Dixit, Grossman and Helpman (1997), Goldberg and Maggi (1997)), combinatorial auction design (e.g., Milgrom (2007)), industrial organization (e.g., Bernheim and Whinston (1989)), and political economy and public finance (e.g., Aidt (1998), Besley and Coate (2001), Persson and Tabellini (2002), Bellettini and Ottaviano (2005), Felli and Merlo (2006)).

Common Agency Games: A General Framework

- N principals, P_i (for $i = 1, ..N$) offer non-cooperatively *contribution schedules* to a common agent.

- The agent may decide (or not) with which subset of principals to contract and how much to produce on their behalf.

- Upon acceptance, the agent chooses an action q (for instance an output level) that may have direct payoff consequences for all players. Payments are enforced.

Examples of Influence Games

- EXAMPLE 1: Public finance: Voluntary contributions to a public good.

  N citizens/public bodies (principals) contribute to finance a public good. A firm/public agency (common agent) pockets the money and produces the good on their behalf.

  Question: Do we reach efficiency under decentralized provision and, if not, why? What forms are taken by the free-riding problem if any?

- EXAMPLE 2: Political economy: Lobbying for a public policy.

  N interest groups (principals) offer political contributions to a policy-maker (common agent) to influence his choice of a tax/subsidy/tariff/... .

  Question: Is the so called *"pluralistic view"* of government (Dahl, 1961), namely that competition between groups reaches efficient political outcomes, a correct approach? If not why?

Examples of Influence Games (2)

- EXAMPLE 3: IO: Split-award auctions.

  Two bidders (principals) bids for shares of a prize. A seller (common agent) decides on how to allocate those shares.

  Question: Does this auction format is efficient ? What are the source of inefficiencies if any? Bidders collusion?

Standard Questions for those Games

In all the above examples, modelers might be interested in both positive and normative issues:

- *The structure of equilibria*: Which of those principals influence what? What are the equilibrium actions? What are the equilibrium contributions?
- *Welfare*: Are those equilibria efficient or *"constrained"* efficient in some sense to be defined? What is the distribution of surplus among players and what are its determinants?

The Model: Conflicting Interest Groups

Two groups, $l = 1, 2$**,** *"horizontal differentiation"***:**

$$u_1(x) = -u_2(x) = -x.$$

$$u_l(x) - T_l.$$

Decision-Maker:

$$\underbrace{u_0(x)}_{\text{Intrinsic/Rest of society}} \quad + \quad \underbrace{\sum_l T_l}_{\text{Transfers received from the groups}} .$$

- $u_0(\cdot)$ strictly concave, single-peaked, symmetric around $x_0 = 0$.
- *Example:* $u_0(x) = -\frac{\beta_0}{2}x^2$.

Efficiency

$$x^* = \arg\max_x -\frac{\beta_0}{2}x^2 + x - x \equiv 0!$$

Truthful Equilibria

Suppose that group 2 offers the non-negative schedule:

$$T_2(x) = \max\{x - C_2, 0\}.$$

Taking only this contract, *DM* would choose:

$$\arg\max_x \max\{x - C_2, 0\} - \frac{x^2}{2} \Rightarrow \text{ Payoff } \max\{0, \frac{1}{2} - C_2\}$$

valid if $1 > C_2$.

Group 1's Best Response

P_1 offers a schedule $T_1(x)$ such that:

- DM's participation constraint is binding:

$$\arg\max_x T_1(x) + \max\{x - C_2, 0\} - \frac{x^2}{2} = \max\{0, \frac{1}{2} - C_2\}.$$

- Maximization of the bilateral payoff $P_1 - DM$

$$\arg\max_x -x + \max\{x - C_2, 0\} - \frac{x^2}{2} \Rightarrow x^* = 0$$

valid if $0 > C_2$.

Generalizations

- Games of congruent interests. More general payoff structures. LeBreton and Laussel (2001).
  More solutions to the fundamental equations.
- The truthful equilibrium is coalition-proof. Bernheim, Whinston and Peleg (1985).

Problems: On the Theory Side

- No asymmetric information. Coasean bargaining of some sorts.
- No instance of free riding.
- No frictions in group building.
- Not a theory of group building.

Problems: On the Political Economy Side

- The distribution of surplus has no impact on policy!
- If there is a problem of control of DM and, if we are under complete information. Then write the optimal decision in the constitution. Done!

Directions:

- Reconciling the theory with a theory of group formation.→ *"When Olson Meets Dahl..."*: FROM INEFFICIENT GROUP FORMATION TO INEFFICIENT POLICY-MAKING. LefÃÍvre and Martimort (2018).

- Reconciling the theory with the difficulty of controlling *DM*. → MENU AUCTIONS AND INFLUENCE GAMES UNDER ASYMMETRIC INFORMATION. Martimort and Stole (201?).

Merging the Two Traditions

"When Olson Meets Dahl...":
From Inefficient Group Formation to Inefficient Policy-Making.
LefÃÍvre and Martimort (2018).

1. GROUPS FORMATION. Two competing groups, composed of **congruent but privately informed** individuals.
   - Groups **appoint lobbyists** (delegation), fix overall contribution, decide how this contribution is shared.
   - → **Within-group free riding.**
2. LOBBYING COMPETITION. Common Agency continuation game between lobbyists → **Endogenous costs and benefits of formation**.

Findings (1): Complete Information: “From Efficient Group Formation to Efficient Policy-Making..".

- Aggregation of preferences **within groups** is efficient. **No free-riding.**
- Each group gets **perfectly** organized **whatever** the “environment". **De facto...**
- **No strategic delegation**. Not so obvious.....
- **Efficient equilibrium of the Common Agency continuation game** ⇒ Aggregation of preferences **across groups** is efficient.

  → The whole process leads to efficient decisions.
  “Dahl is right if Olson is wrong...”

A Number of Puzzles

- Frictionless group formation. **Not "Olsonian"**

- No impact of group formation on policy. *Dichotomy* between formation and competition.

  Unless entry costs into the lobbying process are introduced. Ex: Mitra (1999), Bombardini (2008). Even so, influence is *"all or nothing"*.

- No impact of group formation on *other groups' own formation*.

- No impact of the distribution of surplus on policy decision-making. → **Weird take-away** for a political economy model.

Findings (2): Asymmetric Information. *"From* **In***efficient Groups Formation to* **In***efficient Policy-Making...*

- Groups **generally fail** to be correctly represented (and **"sometimes"** fail to be represented at all).

  Costs and benefits are **endogenous.**

  The distribution of surplus **impacts** on actual policy decision-making.

- **Strategic interactions between groups** arise at the formation stage.

  Informational frictions for all groups are determined altogether at equilibrium...

- Inefficiencies in group formation **percolate** to inefficiencies in policy implementation.

The Model: Interest Groups

Two groups, $l = 1, 2$, of sizes $N_l \geq 2$, *"horizontal differentiation"*:

$$u_1(x) = -u_2(x) = -x.$$

Preferences of individuals within a group are non-observable, *"vertical differentiation"*.

- Private information on preference parameter α_l^i:

$$\frac{\alpha_l^i}{N_l} u_l(x) - t_i.$$

- α_l^i distributed on $\Omega_l \equiv [\underline{\alpha}_l, \overline{\alpha}_l]$, cdf F_l, density $f_l > 0$. $l = 1, 2$.
  - $\underline{\alpha}_l \geq 0$.
  - *Monotone Hazard Rate Property*: $\frac{1-F_l}{f_l}$ is decreasing.
  - Notation: $\boldsymbol{\alpha}_l = (\alpha_l^1, \ldots, \alpha_l^{N_l})$.

The Model: Decision-Maker

$$\underbrace{u_0(x)}_{\text{Intrinsic/Rest of society}} + \underbrace{\sum_l T_l}_{\text{Transfers received from lobbyists}}.$$

- $u_0(\cdot)$ strictly concave, single-peaked, symmetric around $x_0 = 0$.
- *Example:* $u_0(x) = -\frac{\beta_0}{2}x^2$.

Lobbying Process: Appointing a Lobbyist

- Each group delegates to a lobbyist the task of influencing the decision-maker:

$$\beta_l u_l(x) - T_l.$$

- Mesure of the degree of efficiency in group formation:

$$\beta_l(\boldsymbol{\alpha}_l) \neq \alpha_l^*(\boldsymbol{\alpha}_l) \equiv \frac{1}{N_l}\sum_{i=1}^{N_l} \alpha_l^i.$$

Lobbying Process: Common Agency (1)

- *Truthful payment schedules* (an equilibrium refinement...)

$$\tilde{T}_l(x) = \max\{\beta_l u_l(x) - C_l, 0\}$$

- The decision-maker *perfectly aggregates preferences of IGs.*

- Decision

$$x(\beta_1, \beta_2) = \arg\max_x \quad \beta_1 u_1(x) + \beta_2 u_2(x) + u_0(x)$$

$$\Rightarrow u_0'(x(\beta_1, \beta_2)) = \beta_2 - \beta_1.$$

Lobbying Process: Common Agency (2)

- **Conflicting interests** $\Rightarrow$ Unique equilibrium payoff (Laussel and Le Breton (2001)).

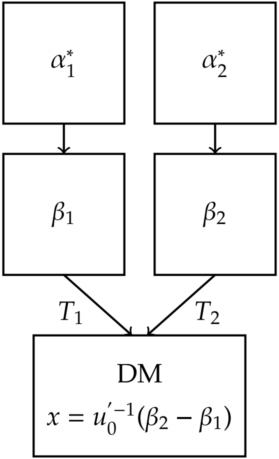
- Equilibrium payments=**Vickrey-Clarke-Groves**!! Pay for the externality that changing the decision exerts on the other lobbyist:

$$T_l(\beta_l, \beta_{-l}) = \beta_{-l}\left(u_{-l}(x(0,\beta_{-l})) - u_{-l}(x(\beta_l,\beta_{-l}))\right)$$

$$+u_0(x(0,\beta_{-l})) - u_0(x(\beta_l,\beta_{-l})).$$

$\Rightarrow$ **Non-manipulable!!** No reason to *strategically delegate* to a lobbyist with a different objective function than the group itself.

$\Rightarrow$ **Sole source of distortion** in the lobbyist's objective function: **Asymmetric information!**


α_1^*
α_2^*
β_1
β_2
T_1
T_2
DM
$x = u_0'^{-1}(\beta_2 - \beta_1)$

Non-Manipulability: Complete Information

Non-manipulability of Clarke-Groves' payments $\Rightarrow$ Group l is *"truthful" (for whatever it means...)*

$$\beta_l(\boldsymbol{\alpha}_l) = \alpha_l^*(\boldsymbol{\alpha}_l) \equiv \frac{1}{N_l}\sum_{i=1}^{N_l} \alpha_l^i.$$

Take-aways:

- Strategy independent of group $-l$'s strategy.
  Clarke-Groves=Dominant strategy
- Competition between lobbyists leads to an efficient decision:

$$x(\alpha_1^*, \alpha_2^*) \equiv x(\beta_1^*(\alpha_1), \beta_2^*(\alpha_2)) \Leftrightarrow u_0'(x(\alpha_1^*, \alpha_2^*)) = \alpha_1^* - \alpha_2^*.$$

- Dahl and Olson on Tango...

Group Formation under Asymmetric Information

MECHANISM DESIGN: No specified intra-group bargaining, *"mediator paradigm"*, encompasses all incentive-feasible possibilities.

Tradition: Myerson and Sattherwaite (1982, bargaining), Laffont and Martimort (1997, collusion in mechanism design).

$$\mathcal{G}_l(\cdot) : \hat{\boldsymbol{\alpha}}_l \longmapsto \mathcal{G}_l(\hat{\boldsymbol{\alpha}}_l) = \left(\beta_l(\hat{\boldsymbol{\alpha}}_l), (t_i(\hat{\boldsymbol{\alpha}}_l))_{i=1}^{N_l}\right)$$

subject to

Incentive Compatibility, *Budget Balance* and *No-Veto constraint*.

Incentive Feasibility

Type α_l^i's expected net payoff from participating in the mechanism: where $\Delta u_l(\beta_l, \beta_{-l}) \equiv u_l(x(\beta)) - u_l(x(0, \beta_{-l}))$.

- *No-Veto*:

$$\mathcal{U}_l(\alpha_l^i) \geq 0 \quad \forall \alpha_l^i \in \Omega_l.$$

- *Incentive compatibility*:

$$\mathcal{U}_l(\alpha_l^i) \max_{\hat{\alpha}_i} \mathbb{E}_{\alpha_{-i}} \left(\frac{\alpha_l^i}{N_l} \mathbb{E}_{\alpha_{-l}} \left(\Delta u_l(\beta_l(\hat{\alpha}_l^i, \boldsymbol{\alpha}_{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l})) \right) - t_i(\hat{\alpha}_l^i, \boldsymbol{\alpha}_{-i}) \right).$$

- *Budget-balance*:

$$\sum_{i=1}^{N_l} t_i(\boldsymbol{\alpha}_l) \geq \mathbb{E}_{\alpha_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) \quad \forall \boldsymbol{\alpha}_l.$$

Incentive Compatibility

Lemma

An allocation $(\mathcal{U}_l(\alpha_l^i), \beta_l(\boldsymbol{\alpha}_l))$ is incentive compatible if and only if:

1. $\mathcal{U}_l(\alpha_l^i)$ *is absolutely continuous, and satisfies the integral representation:*

$$\mathcal{U}_l(\alpha_l^i) = \mathcal{U}_l(\underline{\alpha}_l) + \int_{\underline{\alpha}_l}^{\alpha_l^i} \mathbb{E}_{\boldsymbol{\alpha}_{-i},\boldsymbol{\alpha}_{-l}} \left(\frac{1}{N_l} \Delta u_l(\beta_l(\gamma_i, \boldsymbol{\alpha}_i), \beta_{-l}(\boldsymbol{\alpha}_{-l})) \right) d\gamma_i;$$

2. $\mathbb{E}_{\boldsymbol{\alpha}_{-i},\boldsymbol{\alpha}_{-l}}(\Delta u_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})))$ *is non-decreasing in* α_l^i.

Incentive Compatibility: Intuition

Consider type α_l^i mimicking a type with a slightly lower willingness to pay $\alpha_l^i - d\alpha_l^i$.

- reduces the group's action to $\beta_l(\alpha_l^i - d\alpha_l^i, \boldsymbol{\alpha}_{-i})$;
- reduces the individual contribution $t_i(\alpha_l^i - d\alpha_l^i, \boldsymbol{\alpha}_{-i})$.

The marginal gain of doing so is roughly:

$$\mathcal{U}_l(\alpha_l^i) - \mathcal{U}_l(\alpha_l^i - d\alpha_l^i) \approx \mathbb{E}_{\boldsymbol{\alpha}_{-i},\boldsymbol{\alpha}_{-l}}\left(\frac{1}{N_l}\Delta u_l(\beta_l(\alpha_l^i - d\alpha_l^i, \boldsymbol{\alpha}_{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right) d\alpha_l^i;$$

No Veto

FOCUS: Individual incentives to participate to the group:

$$\mathcal{U}_l(\underline{\alpha}_l) \geq 0.$$

ALTERNATIVE ASSUMPTIONS:

- Only key individuals might have veto power.
- Group blocking.

Incentive Feasibility

Define *"Virtual Parameter"* and *"Average Virtual Parameter"*:

$$h_l(\alpha_l^i) = \frac{1}{N_l}\left(\alpha_l^i - \frac{1 - F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right); \quad h_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_l^i).$$

Proposition

1. *The virtual net gain is non-negative:*

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(h_l^*(\alpha_l)\Delta u_l(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l})) - T_l(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l}))\right) \geq 0.$$

$$\Leftrightarrow$$

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x) + h_l^*(\boldsymbol{\alpha}_l)u_l(x) + \beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right) \geq 0. \quad (1)$$

2. $\mathbb{E}_{\boldsymbol{\alpha}_{-i},\boldsymbol{\alpha}_{-l}}(\Delta u_l(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l})))$ *is non-decreasing in* α_l^i.

The role of informational rents (within the group) as a (variable) cost from forming → *Ramsey-like* analysis soon to come....

Existence of an Efficient Equilibrium $\beta_l(\boldsymbol{\alpha}) = \alpha_l^*(\boldsymbol{\alpha}_l)$ under Asymmetric Information?

Proposition

An efficient equilibrium exists if and only

$$\underbrace{\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\Big[u_0(x) + \alpha_l^*(\boldsymbol{\alpha}_l)u_l(x) + \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})u_{-l}(x))\Big]_{x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}^{x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}\right)}_{\textit{Overall gains from group } l\textit{'s formation}} \tag{2}$$

$$\geq \underbrace{\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\frac{1}{N_l}\left(\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)\Delta u_l(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))\right)}_{\textit{(Variable) Cost of information rents within the group}}.$$

Interpretation

$\rightarrow$ RHS= **Cost of inducing preferences revelation**: A fundamental difference with complete information.

With complete information, $RHS \equiv 0$ and the condition of course holds!

Asymmetric information $\Rightarrow$ Less optimistic view of *"pluralistic politics."*

Information rents within group may preclude representation.

Large Groups, $N_l \to +\infty$ (1)

Strong Law of Large Numbers:

$$\alpha_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} \alpha_l^i \underset{N_l \to +\infty}{\overset{a.s.}{\longrightarrow}} \alpha_l^e = \int_{\underline{\alpha}_l}^{\overline{\alpha}_l} \alpha_l^i f_l(\alpha_l^i) d\alpha_l^i = \text{ Average valuation.}$$

and

$$h_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_l^i) \underset{N_l \to +\infty}{\overset{a.s.}{\longrightarrow}} \underline{\alpha}_l = \int_{\underline{\alpha}_l}^{\overline{\alpha}_l} h_l(\alpha_l^i) f_l(\alpha_l^i) d\alpha_l^i = \text{ Minimum valuation.}$$

Inefficiency in Large Heterogenous Groups

Proposition

Free Riding in large heterogenous groups. *Suppose that* $\underline{\alpha}_l = 0 < \alpha_l^e$. *No efficient equilibrium exists when* N_l *is large enough.*

Proof: Taking limits with $\underline{\alpha}_l = 0$...

$$\mathbb{E}_{\alpha_{-l}}(u_0(x^*(\alpha_l^e, \alpha_{-l}^*)) + \underline{\alpha}_l u_l(x(\alpha_l^e, \alpha_{-l}^*)) + \alpha_{-l} u_{-l}(x(\alpha_l^e, \alpha_{-l}^*)))$$

$$< \mathbb{E}_{\alpha_{-l}}(u_0(x^*(0, \alpha_{-l}^*)) + \underline{\alpha}_l u_l(x(0, \alpha_{-l}^*)) + \alpha_{-l} u(x(0, \alpha_{-l}^*))).$$

Intuition: Each individual minimizes his own contribution. *"Behaves as if he takes as given whatever decision is induced by the contribution of his own group"...*

$\Rightarrow$ Zero contribution overall!

Impossibility to shift the decision towards $x(\alpha_l^e, \alpha_{-l}^*)$.

Efficiency in Large Homogenous Groups

Proposition

Efficient equilibria in large homogenous groups. *Suppose that $\alpha_l^e - \underline{\alpha}_l$ is small enough and $\underline{\alpha}_l > 0$. An efficient equilibrium exists when N_l is large enough.*

Olson: *"Size as an obstacle to efficient formation."*

Not true here: *Size and heterogeneity* **both** *matter.*

Groups of Finite Size and Inefficiencies

- **An equilibrium in mechanisms** for group formation: Myerson (1982), Martimort (1996).

$$\text{Group } l\text{'s objective: } \max_{\mathcal{U}_l(\cdot),\mathcal{G}_l(\cdot)} \sum_{i=1}^{N_l} \mathbb{E}_{\alpha_l^i}(\mathcal{U}_l(\alpha_l^i))$$

subject to incentive feasibility

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x)+h_l^*(\boldsymbol{\alpha}_l)u_l(x)+\beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right)\geq 0 \qquad (3)$$

- **Ramsey-like analysis:** λ_l = Lagrange multiplier for (3).

Inefficient Best Responses in Group Formation

Proposition

INEFFICIENT BEST RESPONSES. *At a best response to group $-l$'s choice $\beta_{-l}(\boldsymbol{\alpha}_{-l})$, group l endows his own lobbyist with a preference parameter $\beta_l^{sb}(\alpha_l)$ such that:*

$$\beta_l^{sb}(\boldsymbol{\alpha}_l) = \max\left\{0, \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i - \frac{\lambda_l}{1+\lambda_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right\}.$$

- If $\lambda_l > 0$, $\beta_l^{sb}(\boldsymbol{\alpha}_l) < \alpha_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i$.

  $\rightarrow$ *"Moderate"* preferences for the lobbyist.

- Informational frictions preclude efficient group formation.

A “Universal" Condition for Inefficiencies, i.e., $\lambda_l > 0$

Assumption

$$\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)} - \mathbb{E}_{\alpha_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)} | \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i = \alpha_l^*\right)$$

is non-increasing in α_l^* $\forall l$.

where Φ_{N_l} (resp. ϕ_{N_l}) is the c.d.f. (resp. density) of $\alpha_l^* = \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i$.

→ Condition easy to check on examples (e.g., uniform).

→ Not a surprise to have conditions on distributions.....

Representation of interests. Strong inefficiency, $\underline{\alpha}_l = 0$

Proposition

When Assumption 1 holds and $\underline{\alpha}_l = 0$:

1. *Group l's best response is always inefficient:*

$$\lambda_l > 0.$$

   $\Rightarrow$ *Frictions!*

2. *Probability that group l has no influence goes to 1 as N_l increases:*

$$\beta_l^{sb}(\boldsymbol{\alpha}_l, N_l) \underset{N_l \to +\infty}{\overset{p}{\to}} 0.$$

Strong Inefficiency of Pluralistic Politics under Asymmetric Information and Finite Sizes

- Groups (almost) always under-represent their interests.

  → **Not a free-riding problem between groups**....but an optimal strategy **within groups**.

- **Extreme inefficiency is possible.**

A Dual Representation of Equilibria

Best-response mapping can be expressed in the *dual space* of Lagrange multipliers :

$$\Lambda_l^*(\lambda_{-l}).$$

Equilibrium (λ_1, λ_2) is thus a pair such that

$$\lambda_l = \Lambda_l^*(\lambda_{-l}) \quad \forall l \in \{1, 2\}.$$

Proposition

Existence of equilibria. *There always exists a (pure strategy) equilibrium.*

In sharp contrast with complete information: **Each group's formation now depends on its competitors!**

Two Opposite Effects

As group $-l$ finds it more difficult to organize itself:

- Less contributions are needed for group l to influence the policy:

  $\rightarrow$ *"Policy shifting" effect.*

  - Internal free-riding becomes less of a curse.

    $\Rightarrow$ *Group l formation easier.*

- Status quo is more favorable to group l:

  $\rightarrow$ *"Status quo" effect.*

  - Less costly not to get organized.

    $\Rightarrow$ *Group l formation more difficult.*

Cross-Effects on Groups Formation

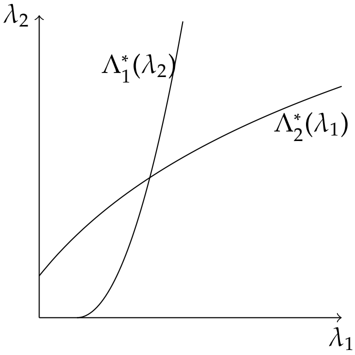
Which effect dominates depends on the decision-maker's preferences $u_0(\cdot)$:

Lemma

The mapping $\Lambda_l^*(\cdot)$ is everywhere non-decreasing (resp. non-increasing) if and only if $u_0'''(\cdot) \geq 0$ (resp. $u_0'''(\cdot) \leq 0$).

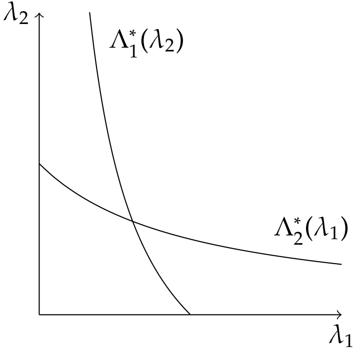
“Game" of formation among groups may exhibit *strategic complementarity* (upward sloping best-response curves in the (λ_1, λ_2) space) or *substitutability* (downward sloping).

Best Response: Complementarity, $u_0'''(\cdot) \geq 0$



As a group manages to get more efficiently organized, it faces a **stronger competitor.** → *"Status quo effect"* dominates.

Best Response: Substitutability, $u_0'''(\cdot) \leq 0$



As a group manages to get more efficiently organized, it faces a **weaker competitor.** → *"Policy shifting effect"* dominates.

Commitment Effects: Towards a Strategic “IO" View of Group Formation

- *Information sharing* within a group

Exemple: Coalitions of IGs themselves in legislative politics, *Strong coalitions* in the sense of Hula (1999):

$\rightarrow \lambda_{-l} = 0$: Commitment to soften competition when $u_0''' \leq 0$.

The *“policy-shifting” effect* exacerbates frictions within group l, while the *“status quo” effect* does the reverse.

When the first effect dominates (i.e., $u_0''' \leq 0$), a group which has solved its own free riding problem can not only buy influence more easily but it also weakens its competitor’s representation.

Illustrations

Coalitions bound by strong ties might thus exclude rivals more easily than what less well-organized groups would do.

- *Illustration 1.* The U.S. sugar industry; the Fanjul brothers. Strong ties to keep price high and a quotas system. Against sugar users (Pespsi, Coca, ...)

- *Illustration 2.* The strong political influence of senescent industries.

  Current explanation: Not enough profit for entrants to pay the fixed cost of lobbying. Hillman (1982), Brainard and Verdier (1997), Baldwin and Robert-Nicoud (2007).

  Alternative/complementary explanation: Strong ties between incumbents knitted over a long period of time maintain entrants out.

Entry Costs

- ENTRY COSTS. Organization or enforcement costs beyond asymmetric information *per se*.

  Let denote by K_l a fixed cost of group formation.

  Incentive feasibility:

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x)+h_l^*(\boldsymbol{\alpha}_l)u_l(x)+\beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right)\geq K_l>0. \tag{4}$$

  Group l's contribution must increase to cover this extra fixed cost. In response, informational frictions are more pronounced.

  $\rightarrow$ The best-response mapping Λ_l^* is shifted upwards, modifying accordingly the set of possible equilibria of the game.

- *Illustration 4.* Bombardini (2008): When the distribution of firm sizes has a "fatter tail," lobbying for protection is more effective.

Welfare : Groups' Payoffs

Both groups as a whole may benefit from informational frictions.

- *Softened competition* at the common agency stage.

Proposition

Consider two symmetric groups: same size $N_1 = N_2 = N$, members' valuations α_l^i drawn from the same uniform distribution on $[0, \overline{\alpha}]$. Assume $u_0(x) = -\frac{\beta_0}{2}x^2$. For N large enough, each group is ex ante better off when group formation takes place under asymmetric information.

Under complete information, payments are wasted to keep the *status quo*....

Decision maker's payoff

Proposition

The decision-maker's ex ante and ex post payoffs are always lower under asymmetric information.

Less competition between groups, lower contributions ⇒ Less rent for the decision-maker.

Asymmetric information can be a rationale for the low contributions obtained in politics. The so called *"Tullock Paradox"*.

Veto Power For Key Players Only

Lobbyist's preferences:

$$\beta_l^{sb}(\boldsymbol{\alpha}_l) = \frac{1}{N_l} \max\left(\sum_{i=1}^{\hat{N}_l} \alpha_l^i + \left\{ \sum_{i=\hat{N}_l+1}^{N_l} \alpha_l^i - \frac{\hat{\lambda}_l}{1+\hat{\lambda}_l} \frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)} \right\}; 0 \right). \qquad (5)$$

$\hat{\lambda}_l$ is the Lagrange multiplier for the new incentive-feasibility condition:

$$\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}} \left(\left[u_0(x) + \tilde{h}_l^*(\boldsymbol{\alpha}_l) u_l(x) + \beta_{-l}(\boldsymbol{\alpha}_{-l}) u_{-l}(x) \right]_{x(0, \beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))} \right) \geq 0 \qquad (6)$$

where $\tilde{h}_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l} \left(\sum_{i=1}^{N_l} \alpha_l^i - \sum_{i=\hat{N}_l+1}^{N_l} \frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)} \right)$.

Mutatis mutandis, same results follow.

Of course, **less inefficiency.**

Free-Riding on Participation

Free riding on participation is prevented when:

$$\mathcal{U}_l(\alpha_l^i) \geq \mathcal{U}_l^r(\alpha_l^i) = \frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u(\beta_l(0,\boldsymbol{\alpha}_l^{-i}),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) \geq 0. \qquad (7)$$

A priori, a more stringent participation constraint. Frictions might now be more significant

Nevertheless, observe that $\mathcal{U}_l^r(0) = 0$ and

$$\dot{\mathcal{U}}_l(\alpha_l^i) - \dot{\mathcal{U}}_l^r(\alpha_l^i) = \frac{1}{N_l}\left(\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u(\beta_l(\alpha_l^i,\boldsymbol{\alpha}_l^{-i}),\beta_{-l}(\boldsymbol{\alpha}_{-l})) - \Delta u(\beta_l(0,\boldsymbol{\alpha}_l^{-i}),\beta_{-l}(\boldsymbol{\alpha}_{-l})))\right.$$

since $\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u(\beta_l(\alpha_l^i,\boldsymbol{\alpha}_l^{-i}),\beta_{-l}(\boldsymbol{\alpha}_{-l}))$ is non-decreasing in α_l^i.

Thus (7) is only binding at $\underline{\alpha}_l = 0$ if it is so. Hence, (7) is no more stringent than the standard veto constraint. When $\underline{\alpha}_l = 0$, our results are thus all robust.

Congruent Groups: $u_1(x) = u_2(x) = x$

- Technical difficulties (payments are not uniquely defined) $\rightarrow$ Selection a sharing-rule: *Shapley Value*.
    - Payments are no longer *VCG* $\Rightarrow$ **Free-riding between groups** in addition to free-riding within-groups.
- Merging coalition of congruent interests entails both costs and benefits.
    - Lower status quo payoffs with a merger. Outside option is zero. $\Rightarrow$ Informational costs may increase with a merger.
    - Free-riding problem across groups internalized.

$\rightarrow$ We provide an example showing that congruent groups may prefer to merge. This justifies our focus on competition between *two* conflicting groups in the first place.

Endogenizing Transaction Costs of Side-Payments

Common assumption: Transaction costs of side-payments.

- Reduced forms for the collective action problems faced by the group.
- Becker (1983), Bernheim and Whinston (1986), Grossman and Helpman (1994), Laffont and Tirole (1993, Chapter 15).

$$\alpha \Delta u - (1 + \mu)T$$

Our model *endogenizes* these costs:

- But type-dependent (α_l) and environment-dependent (λ_l).
- More complex than reduced-form models.→ *Informational transaction costs:*

Conclusion

- A model putting together group's formation and lobbying competition.
- Free-riding within group precludes efficient formation.→ Pessimistic view of pluralistic politics.
- The model can be viewed as a response to a number of existing puzzles in the lobbying (common agency) literature:
  - Frictionless formation. **Not here.**
  - No impact of group formation on policy. *Dichotomy* between formation and competition. **Not here.**
  - No impact of group formation on *other groups formation*. **Not here.**
  - No impact of the distribution of surplus on actual policy decision-making. → weird as a model of political economy. **Not here.**

Menu Auctions and Influence Games under Asymmetric Information: A General Framework (2)

COMPLETE INFORMATION: Huge literature. More on this later.

ASYMMETRIC INFORMATION: A particular source of inefficiency on which we focus in this paper.

- The agent is privately informed on his type θ at the time of contracting. Principals ignore this parameter.
- *Examples:* Cost of producing the public good/Political opportunity costs/Value of the award.

Taxonomy

- **Delegated common agency:** The agent may *a priori* accept any subset of contracts and reject others.

  Examples:

  - Menu auctions, Bernheim and Whinston (1986);
  - Lobbying games, Grossman and Helpman (1994), Grossman, Helpman and Dixit (1997) and others.

  The focus is on the distribution of payoffs under complete information (general analysis in Laussel and Lebreton, 1998, 2001).

- $\neq$ **Intrinsic common agency:** The agent must either accept or reject all contracts together.

  Examples:

  - Multiple regulators (PUC, EPA) controlling a regulated firm both for its price and pollution, Baron (1985);
  - Privatization, Laffont and Tirole (1993).
  - Martimort, Semenov, Stole (2018, TE): *"A Complete Characterization*

Taxonomy (2)

Principals influence the agent's choice via contracts which condition payments on actions.

Two variations of the model:

- **Private action:** The action is a vector

  $$q = (q_1, ..., q_n)$$

  Principal i offers a schedule $t_i(q_i)$ only.

  Examples:

  - Competitive nonlinear pricing, Martimort (1992), Stole (1992)Mezzetti (1997), Martimort and Stole (2009);
  - Manufacturers/common retailer relationships, Martimort (1996), Calzolari and De Nicolo (2013,15);
  - Financial markets, Biais, Martimort and Rochet (2000);
  - International taxation for MNEs, Olsen and Osmuddsen (2003).

Taxonomy (3)

- **Public actions:** Each principal can make payments conditional on all components of the agent's choice. The simple case where the private information parameter $\theta \in \Theta$ is one-dimensional like the public action (screening device) $q \in \mathcal{Q} \subseteq \mathbb{R}^1$.

  Principal i offers a schedule $t_i(q)$.

  Examples:

  - Lobbying games.
  - Multiple regulators (PUC, EPA) controlling a firm's output both for its impact on price and pollution, Baron (1985);
  - Privatization, shareholders and regulators controlling the firm's profit, Laffont and Tirole (1993).

Our Focus

- DELEGATED AGENCY GAMES in which principals contract on PUBLIC ACTIONS under asymmetric information.

Objectives of the paper

- CHARACTERIZING EQUILIBRIUM ALLOCATIONS and their properties (existence, inefficiency, nature of the output distortions and their directions, range of equilibrium outputs).
- DETERMINING WHEN A GIVEN PRINCIPAL IS "ACTIVE" (i.e., whether he contributes a positive amount and for which subset of the agent's types). Do we have overlapping activity sets or not and under which circumstances?
- FINDING SOME GENERAL PROPERTIES OF CONTRIBUTION SCHEDULES.

Complete information: What do we know?

Seminal paper: Bernheim and Whinston (1986).

- RESULT 1: Decentralized bargaining under common agency yields an efficient outcome. A version of the Coase Theorem with bilateral contracting.

$$q(\theta) = \arg\max_{q \in Q} \sum_{i=0}^{N} S_i(q) - \theta q.$$

- RESULT 2: Implementation by means of *"truthful schedules"*.

$$t_i(q) = \max\{S_i(q) - C_i, 0\}.$$

Why Do We Get Those Results?

At a best response, each principal designs a contract so as to

- Maximize bilateral surplus with the agent.... taking as given contributions by others.
- Extract as much as possible of the agent's surplus; up to his next best opportunity which consists in contracting with other principals.
- A truthful schedule is *always* in the best-response correspondence.

Asymmetric information: Computing best responses

BILATERAL CONTRACTING AND ASYMMETRIC INFORMATION. Implications on the rent-efficiency trade-off reached in each bilateral relationship:

1. BILATERAL EFFICIENCY does not mean overall efficiency since contributions by other principals are not necessarily truthfulÂă under asymmetric information.
2. BILATERAL EFFICIENCY itself is lost....Incentive constraints within each bilateral contract.

3. ENDOGENOUS TYPE-DEPENDENT PARTICIPATION CONSTRAINTS depend also on the contributions of others (delegated agency). Participation constraints limit the capacity to extract rent in any given bilateral relationship. → Definition of the *activity sets*.

Asymmetric information: What do we get?

- COMPOUNDING INEFFICIENCIES: Whether different principals induce similar distortions or not depend on whether they have *congruent* or *conflicting* preferences.

- ACTIVITY SETS: Complex to determine a priori unless principals' surplus functions are linear in the decision variable ($S_i(q) = s_i q$). Activity sets then have a simple *"cut-off" structure.*

  In general, *inefficient "representation"* with some inactive principals being left out of the bargaining process.

- NON-TRUTHFUL DISTORTIONS: Marginal contributions = **"virtual" marginal valuations ≠ marginal valuations.**

$$\bar{t}_i(q) = \int_{\bar{q}(\hat{\theta}_i)}^{q} \max\left\{s_i - \frac{F(\vartheta(q))}{f(\vartheta(q))}, 0\right\} dq, \text{ if } \frac{1}{f(\bar{\theta})} \geq s_i > 0, \quad (8)$$

$$\bar{t}_i(q) = \int_{\bar{q}(\hat{\theta}_i)}^{q} \min\left\{s_i + \frac{1 - F(\vartheta(q))}{f(\vartheta(q))}, 0\right\} dq \text{ if } -\frac{1}{f(\underline{\theta})} \leq s_i < 0. \quad (9)$$

For our specific examples: What do we get?

- EXAMPLE 1: Voluntary contributions to a public good.

  Principals are *congruent*.

  Under asymmetric information, free-riding takes two forms:

  - *Intensive margins.* Lower contributions for a given coalition of “active" contributors.
  - *Extensive margins.* Less contributors than if they had cooperated.

- EXAMPLE 2: Lobbying for a public policy with conflicting interest groups influencing a policy-maker.

  Principals are *conflicting*.

  - Either *segmented* or *overlapping* influences.

Road map

1. THE MODEL.

2. BEST RESPONSES: (necessary/sufficient conditions).

3. SOME GENERAL RESULTS ON CHARACTERIZATION.

4. LINEAR SURPLUSES:

    - AN "AGGREGATE" FORMULATION available for both continuous (so-called **"virtually truthful"**) and discontinuous equilibria.

    - EXAMPLE 1: Voluntary contributions to finance a public good.

    - EXAMPLE 2: Lobbying games.

The model

PREFERENCES, INFORMATION, STRATEGIES.

$$\text{Agent:} \quad U = S_0(q) - \theta q + \sum_{i=1}^{n} t_i,$$

$$\text{Principal } i\text{:} \quad V_i = S_i(q) - t_i,$$

- The benefit functions $S_0, \ldots, S_i, i \in N$, are upper semi-continuous.
- The agent has private information about $\theta \in \Theta$, cumulative distribution function, $F : \Theta \to [0, 1]$, and bounded density function $f : \Theta \to \Re_{++}$.
- "Generalized monotone hazard rate" property; i.e., $\frac{F(\theta)-\kappa}{f(\theta)}$ is strictly increasing over Θ for any $\kappa \in [0, 1]$.

The model, continued

- Any first-best optimal allocation,

$$q^{FB}(\theta) \in \arg\max_{q \in Q} \sum_{i=0}^{n} S_i(q) - \theta q,$$

is **minimally** separating; i.e.,

$$q^{FB}(\overline{\theta}) < q^{FB}(\underline{\theta}).$$

Timing

1. The agent learns θ.
2. Principals offer non-cooperatively contracts $t_i(q) \geq 0$ (upper semi-continuous).
3. The agent accepts any subset of those offers, chooses q and receives the corresponding payments $t_i(q)$ for those accepted offers.

Equilibrium

DÃľfinition

An equilibrium is a profile of principals' contribution schedules, $\bar{t} = (\bar{t}_1, \ldots, \bar{t}_n)$, and an agent action's strategy, $\bar{q}_0(\theta|t)$, such that the following properties hold.

1. *Given any profile of non-negative contributions $t = (t_1, \ldots, t_n)$,*

$$\bar{q}_0(\theta|t) \in \arg\max_{q \in Q} S_0(q) - \theta q + \sum_{i \in N} t_i(q).$$

2. *$\bar{t}_i$ maximizes principal i's expected payoff given the other principals' aggregate contribution schedule $\bar{t}_{-i}$:*

$$\bar{t}_i \in \arg\max_{t_i \in \mathcal{T}} \int_{\Theta} (S_i(\bar{q}_0(\theta|t_i, \bar{t}_{-i})) - t_i(\bar{q}_0(\theta|t_i, \bar{t}_{-i})))f(\theta)d\theta.$$

Incentive feasible set

The agent's surplus from contracting with all principals:

$$U(\theta) = \max_{q \in Q} \; S_0(q) - \theta q + t_i(q) + T_{-i}(q)$$

where $T_{-i}(q) = \sum_{j \neq i} \bar{t}_j(q)$.
The agent's surplus from contracting with all principals but i:

$$\overline{U}_{-i}(\theta) \equiv \max_{q \in Q} \; S_0(q) - \theta q + T_{-i}(q).$$

Lemma

A rent-output profile (U, q) is implementable by principal P_i given the equilibrium transfer profile $\bar{t}_{-i}$ if and only if

$$U(\theta) \geq \overline{U}_{-i}(\theta) \text{ for all } \theta \in \Theta, \tag{10}$$

$$-q(\theta) \in \partial U(\theta) \text{ for all } \theta \in \Theta, \tag{11}$$

$$U \text{ is convex.} \tag{12}$$

Activity sets

DÃl'finition

Principal i's **equilibrium activity set** *is defined as*

$$\overline{\Theta}_i = \left\{\theta \in \Theta \,|\, U(\theta) > \overline{U}_{-i}(\theta)\right\},$$

and the set of active principals for θ is given by the **equilibrium activity map**

$$\overline{\alpha}(\theta) \equiv \left\{i \in N \,|\, U(\theta) > \overline{U}_{-i}(\theta)\right\}.$$

REMARK: Principal i's contribution is not necessarily zero at a boundary of the activity set but yet the participation constraint is binding.

This would indeed be the case when P_i “becomes" an active principal with a positive transfer at that point. Inducing a jump discontinuity in action.

A non-smooth control problem

Define the overall benefit of producing q for the pair made of principal i and the agent as:

$$W_i(q, t_{-i}) \equiv S_i(q) + S_0(q) + t_{-i}(q).$$

We now state the relevant program for principal i when facing the profile $t_{-i} = \sum_{j \neq i} t_j$:

$$(\mathcal{P}_i) : \max_{(U,q)} \int_\Theta (W_i(q(\theta), t_{-i}(q(\theta))) - \theta q(\theta) - U(\theta)) f(\theta) d\theta$$

subject to (10), (11), (12).

Optimality conditions

Proposition

Given the profile of transfers offered by rival principals, t_{-i}, the rent-ouput profile $(\overline{U}, \overline{q})$ is a solution to $(\mathcal{P}_i)$ if and only if $(\overline{U}, \overline{q})$ satisfies (10), (11), (12) and there exists a probability measure μ_i (possibly with mass points) with an associated adjoint function, $\overline{M}_i : \Theta \to [0,1]$, defined by $\overline{M}_i(\underline{\theta}) = 0$ and for $\theta > \underline{\theta}$,

$$\overline{M}_i(\theta) \equiv \int_{[\underline{\theta},\theta)} \mu_i(d\theta),$$

such that the following two conditions are satisfied:

$$\operatorname{supp}\{\mu_i\} \subseteq \overline{\Theta}_i^c \equiv \left\{\theta \,|\, \overline{U}(\theta) = \overline{U}_{-i}(\theta)\right\}, \tag{13}$$

$$\overline{q}(\theta) \in \arg\max_{q \in \mathcal{Q}} \; W_i(q, t_{-i}(q)) + \left(\frac{\overline{M}_i(\theta) - F(\theta)}{f(\theta)} - \theta\right) q, \; a.e.\; \theta \in \Theta. \tag{14}$$

Methodological contribution

- Minimal restrictions on *"smoothness"* of schedules to avoid restrictions on equilibria$\Rightarrow$ Need for powerful tools from optimal control in non-smooth analysis (Clarke 1980, Vinter 2004).

- Contribution schedules could *"jump"* to attract the agentÂŠ's services. Discontinuities of equilibrium output might not be so unexpected?

  Example 2: Conflicting lobbyists fighting to induce a decision-maker to go on either side of the political spectrum.

- Some equilibria are discontinuous. Assuming continuity would fail to unveil those equilibria.

Methodological contribution (2)

→ Technical tools developed in Martimort and Stole (2017), *"Participation Constraints in Discontinuous Adverse Selection Models"*.

REMARK 1: Jullien (2000) requires C^2 data. Too much in a common agency context, we want instead to derive endogenously smoothness properties of contribution schedules (if any....) with much *no a priori* restriction. *Non-smooth optimal control.*

REMARK 2: Sufficiency comes from adapting Arrow conditions in a non-smooth context.

Virtual costs $\theta + \frac{F(\theta)-\overline{M}_i(\theta)}{f(\theta)}$

- *Incentive distortions* are captured by the usual hazard rate term $F(\theta)/f(\theta)$ which reduces output.
- *Participation distortions* increase output on any activity sets. The new non-negative term $\overline{M}_i(\theta)/f(\theta)$.
    - When $\overline{M}_i(\theta) < F(\theta)$. Principal i finds it cheap to influence types below θ. *"Best type"* for principal i is $\underline{\theta}$.
    - When $\overline{M}_i(\theta) > F(\theta)$. Principal i finds it too costly to induce participation from types less than θ with possibly some inactivity over some range. *"Best type"* for principal i is $\bar{\theta}$.

More properties

1. $\overline{M}_i$ is constant over any connected interval of $\overline{\Theta}_i$;

2. if $S_i : Q \to \mathfrak{R}$ is differentiable and concave, then over any open interval of $\overline{\Theta}_i^c$ for which $\overline{q}_{-i}(\cdot)$ is strictly decreasing,

$$\overline{M}_i(\theta) = F(\theta) - S_i'(\overline{q}_{-i}(\theta))f(\theta); \tag{15}$$

Structure of equilibria

Aggregating the best-response conditions across all principals $i \in \overline{\alpha}(\theta) \subseteq N$ for each $\theta \in \Theta$

$\Rightarrow$ Characterization of equilibrium output (an *"aggregate"*) as a solution to a *self-generating* maximization problem. Such maximization problem depends on its solution. Funny: Keep the fixed-point nature of equilibrium but reduces the analysis to that of a single optimization problem instead of a collection of n such problems.

$\rightarrow$ Martimort and Stole (2012, GEB) *"Representing Equilibrium Aggregates in Aggregate Games with Applications to Common Agency"*.

Structure of equilibria (2)

Theorem

$$\bar{q}(\theta) \in \arg\max_{q \in Q} \; S_0(q) - \theta q + \sum_{i=1}^{n} S_i(q) + \sum_{i=1}^{n} \left(\frac{\overline{M}_i(\theta) - F(\theta)}{f(\theta)} \right) q + (n-1)\left(S_0(q) - \theta q + \overline{T}(q) \right), \; a.e. \quad (16)$$

$$\bar{q}(\theta) \in \arg\max_{q \in Q} \; S_0(q) - \theta q + \sum_{i \in \overline{\alpha}(\theta)} S_i(q) + \sum_{i \in \overline{\alpha}(\theta)} \left(\frac{\overline{M}_i(\theta) - F(\theta)}{f(\theta)} \right) q + (|\overline{\alpha}(\theta)| - 1)\left(S_0(q) - \theta q + \overline{T}(q) \right), \; a.e. \quad (17)$$

Still lots of work to do....

GETTING RID OF THE TERM

$$(|\overline{\alpha}(\theta)| - 1)\left(S_0(q) - \theta q + \overline{T}(q)\right),$$

→ *If we could differentiate...* Use the *"Agent's first-order condition"/Envelope Condition....*

SUFFICIENCY.... Means reconstruct the equilibrium transfers.

FINDING THE ACTIVITY SETS, OR/AND THE ADJOINT FUNCTIONS

$$\overline{M}_i(\theta).$$

→ Specify preferences....

Common-agency games with linear payoffs. Virtual surplus

DÃl'finition

A delegated common-agency game is linear if

- *for each* $i \in N$, $S_i(q) = s_i q$ *with* $s_i f(\bar{\theta}) < 1$ *if* $s_i \geq 0$ *or* $-s_i f(\underline{\theta}) < 1$ *if* $s_i < 0$
- S_0 *is concave.*

Games with linear payoffs. "Virtual surplus"

Partition principals into two sets, $\mathcal{A} = \{i \in N \mid s_i > 0\}$ and $\mathcal{B} = N \setminus \mathcal{A}$.

$$\text{“VIRTUAL (MARGINAL) SURPLUS"} \equiv \beta_i(\theta) \equiv \begin{cases} \max\left\{s_i - \frac{F(\theta)}{f(\theta)}, 0\right\} & i \in \mathcal{A}, \\ \min\left\{s_i + \frac{1-F(\theta)}{f(\theta)}, 0\right\} & i \in \mathcal{B}, \end{cases}$$

$$\text{“AGGREGATE VIRTUAL (MARGINAL) SURPLUS"} \equiv \beta(\theta) \equiv \sum_{i \in N} \beta_i(\theta).$$

Games with linear payoffs. *The "adjoint functions" part...*

The following adjoint functions support any non-degenerate equilibrium allocation, $\overline{q}$:

$$\overline{M}_i(\theta) = \max\{F(\theta) - s_i f(\theta), 0\}, \quad i \in \mathcal{A},$$

$$\overline{M}_i(\theta) = \min\{F(\theta) - s_i f(\theta), 1\}, \quad i \in \mathcal{B}.$$

IMPORTANT STEP OF THE ANALYSIS$\Rightarrow$ It "separates" the analysis of activity sets from the analysis of distortions.

Modified Lindhal-Samuelson condition.

Theorem

Suppose that the common-agency game is linear. If $\overline{q}$ is an equilibrium allocation, then

$$\overline{q}(\theta) \in \arg\max_{q \in \overline{q}(\Theta)} S_0(q) + (\beta(\theta) - \theta)q, \textit{ for all } \theta \in \Theta. \tag{18}$$

Moreover, if an allocation $\overline{q}^{Q}(\theta)$ satisfies

$$\overline{q}^{Q}(\theta) \in \arg\max_{q \in Q} S_0(q) + (\beta(\theta) - \theta)q, \quad \textit{for all } \theta \in \Theta, \tag{19}$$

then $\overline{q}^{Q}$ is an equilibrium allocation. Such an equilibrium allocation always exists.

Some remarks

REMARK 1: All endogenous variables related to activity sets and equilibrium transfers have now disappeared.

REMARK 2: Links with the literature on mechanism design without transfers (Melumad and Shibano, 1991, Alonso and Matousheck, 2008).

Everything happens as if the equilibrium output $\overline{q}$ and its range $\overline{Q}$ were chosen by *a surrogate principal* who aggregates the behavior of all principals and maximizes the *aggregate virtual surplus.*

REMARK 3: Choosing $\overline{q}^{\mathcal{Q}}(\theta)$ is akin to an *equilibrium refinement*$\longrightarrow$ $\mathcal{Q}$-Maximal equilibrium.

Q-Maximal equilibrium allocations

Each principal “shades" his marginal valuation for the agent’s marginal action:

$$\bar{t}_i'(q) = \beta_i(\overline{\vartheta}(q)) = \begin{cases} \max\left\{s_i - \frac{F(\overline{\vartheta}(q))}{f(\overline{\vartheta}(q))}, 0\right\}, & i \in \mathcal{A}, \\ \\ \min\left\{s_i + \frac{1-F(\overline{\vartheta}(q))}{f(\overline{\vartheta}(q))}, 0\right\}, & i \in \mathcal{B}, \end{cases}$$

where $\overline{\vartheta}(q)$ is the appropriate inverse of $\overline{q}(\theta)$.

→ Natural generalization of Bernheim and Whinston’s (1986) truthful equilibria in games of complete information.

Other (non-maximal) equilibrium allocations. Preliminaries

1. All equilibrium allocations within the interior of the equilibrium range are equal to the maximal allocation.
2. Even when S_0 is strictly concave and the maximal allocation $\overline{q}^Q$ satisfying (19) is necessarily continuous, an equilibrium allocation which satisfies only (18) may exhibit discontinuities at θ_0. Then $q_1 = \overline{q}(\theta_0^+)$ and $q_2 = \overline{q}(\theta_0^-) > q_1$ must ensure *continuity of the surrogate principal's virtual surplus*:

$$S_0(q_1) + (\beta(\theta_0) - \theta_0)q_1 = S_0(q_2) + (\beta(\theta_0) - \theta_0)q_2. \qquad (20)$$

3. Bunching on both sides of the discontinuity.

Other (non-maximal) equilibrium allocations. More...

Theorem

Let S_0 be strictly concave and $\hat{q} \in \operatorname{int} \overline{q}^Q(\Theta)$ be an interior action at which all principals are active in the maximal equilibrium. If all principals' preferences are congruent, then there exist a sufficiently small neighborhood (q_1, q_2) of $\hat{q}$ such that the allocation

$$\overline{q}(\theta) \in \arg \max_{q \in \overline{q}^Q(\Theta) \setminus (q_1, q_2)} S_0(q) + (\beta(\theta) - \theta)q, \tag{21}$$

is also an equilibrium allocation. If the agent's type is uniformly distributed, then the conclusion of (21) holds regardless of whether preferences are congruent or opposed.

In addition, every type agent weakly prefers the maximal equilibrium $\overline{q}^Q$ to $\overline{q}$, i.e., $\overline{U}^Q(\theta) \geq \overline{U}(\theta)$, with strict preference for some positive measure of types.

Other (non-maximal) equilibrium allocations. Comments

A “naive" approach assuming twice continuously schedules would fail to uncover those non-maximal equilibria....

The “price" is NON-SMOOTH ANALYSIS...

Application 1: Voluntary contributions with congruent principals

In the public goods game with quadratic preferences for the agent, $S_0(q) = -\frac{1}{2}q^2$, and $s_1 > s_2$, the virtually-truthful equilibrium allocation is

$$\bar{q}(\theta) = \max\left\{\sum_{i\in N} \max\left\{s_i - \frac{F(\theta)}{f(\theta)}, 0\right\} - \theta, 0\right\}. \tag{22}$$

TWO SORTS OF INEFFICIENCIES:

- **"Double distortion"** when both principals are active. Same as under intrinsic common agency.
- **Free-riding in participation** by the "weakest" principal. Specific to delegated agency.

→ **Endogenous exclusivity.**

Voluntary contributions to a public good (2).

Voluntary contributions to a public good (3): Remarks.

REMARK 1: Exclusivity here ≠ exclusivity in Bernheim and Whinston (1998, JPE) who instead model a setting with *ex ante* contracting. Here, participation constraints are *ex post*.

Exclusion over some ranges of cost realizations.

REMARK 2: Keeping $\sum_{i=1}^{2} s_i$ constant, the equilibrium output may change. Asymmetry induces exclusion.

→ **Non-neutrality** reminiscent of the public finance literature: See Bergstrom, Blume and Varian (1986, JPubE) among others.

Application 2: Influence and lobbying games

Two competing interest groups having instead conflicting preferences $S_1(q) = -S_2(q) = q$.

The decision-maker (agent) has some ideal policy at $-\theta$ and $S_0(q) = -\frac{q^2}{2}$.

Symmetric distribution over $[-\delta, \delta]$ with $\delta < 1$.

Complete Information. Efficient decision, $q^*(\theta) \equiv -\theta$. Contributions are wasted to keep such balance!

Influence and lobbying games (2)

Proposition

The maximal equilibrium allocation of the lobbying game is

$$\bar{q}(\theta) = \max\left\{1 - \frac{F(\theta)}{f(\theta)}, 0\right\} + \min\left\{-1 + \frac{1 - F(\theta)}{f(\theta)}, 0\right\} - \theta. \qquad (23)$$

If θ is uniformly distributed, the activity sets of the principals are

$$\overline{\Theta}_1 = [-\delta, \min\{1-\delta, \delta\}) \quad \textit{and} \quad \overline{\Theta}_2 = (\max\{\delta - 1, -\delta\}, \delta]. \qquad (24)$$

If $\delta < 1$, COMMON INFLUENCE on intermediate types.

Otherwise, SEPARATE DOMAINS of influence.

Influence and lobbying games (3)

Influence and lobbying games (4)

- **Both principals are active** in the middle. Contributions just added up to push decisions in opposite directions.

- **Unchallenged influence** on extremes. *"Buy who is close to you."*

- **Small contributions** compared with complete information → A possible answer to the *small money in politics* puzzle.

Discontinuous equilibria

Proposition

Suppose that $s_1 = -s_2 = 1 < 2\delta$, $S_0(q) = -\frac{q^2}{2}$ *and that* θ *is uniformly distributed on* $\Theta = [-\delta, \delta]$*. For any* $q_0 \in (0, (1-\delta)\sqrt{3}]$*, there exists an equilibrium with a discontinuity at* $\theta_0 = 0$ *and such that* $\bar{q}(0^-) = -\bar{q}(0^+) = q_0$*. Both the agent's rent and the principals' expected payoffs in such discontinuous equilibria are lower than at the maximal equilibrium.*

The “modified" tariffs:

$$\bar{t}_i(q) = \begin{cases} 0 & \text{for } q \in (-q_0, q_0), \\ \bar{t}_i^{Q}(q) & \text{otherwise,} \end{cases}$$

support the discontinuous equilibrium.

Discontinuous equilibria. But no *head-to-head* competition.

Conclusion

- METHODOLOGICAL CONTRIBUTION. General framework to derive equilibrium properties without too much restrictions on schedules.
- APPLICATIONS illustrate the richness of delegated common agency games under asymmetric information. Rich patterns of inefficiencies and contributions. Agnostic about equilibrium selection.
- Still quite tractable analysis.
- Congruent and conflicting principals introduce different distortions. The focus is no longer on how principals share the grand-coalition surplus but on their respective impact on distortions.

Thank you