A Model of Certification Effect in Venture Capital

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Abstract

This paper presents a model of informational monopoly in venture capital. An informed leader agency knows the exact success probability of each deal it sources and proposes a financing mechanism to outside investors. Under blind-pool contracts—where investors commit capital before the agency reveals deal quality—the agency achieves the social optimum by hiding private information until execution, fully extracting the investors' expected surplus through the competitive structure of the investor market. A uniform allocation is an optimal solution in this regime, though not the unique one. Under deal-by-deal contracts—where the agency reveals deal quality upfront—the IC and pointwise IR constraints jointly force the optimal allocation to be uniquely uniform, but surplus transfer to investors and inefficient exclusion of positive-NPV projects arise. Investors cannot unilaterally impose deal-by-deal contracts without sacrificing social welfare.

JEL Codes: G24, D82, L12

Keywords: Venture Capital, Mechanism Design, Information and Knowledge, Monopolization Strategies

1 Introduction

Venture capital markets are characterized by a rich ecosystem of actors who differ both in the *degree* of information they hold and in the *stage* at which they intervene. Some investors specialize in early screening—accelerators such as Y Combinator or Techstars select and mentor startups intensively before any external financing occurs (Hallen et al., 2014, 2020). Others intervene at later stages, relying partly on the track record and co-investment decisions of earlier entrants as a certification signal. A common pattern in this ecosystem is a *sequencing* structure: one actor leads the financing round and sets the contractual terms, while others commit capital on the basis of the leader's track record. The extent to which followers rely on the leader's assessment rather than on their own due diligence varies across deals and stages, but the asymmetry is a recurring feature of VC markets.

A striking illustration of this structure is the LP–GP relationship in private equity. Lerner et al. (2022) document that 93% of LP–GP contracts are *blind-pool* contracts: limited partners commit capital before the GP has identified any specific deals. The remaining 7% are *deal-by-deal* contracts, in which LPs retain project-level approval rights. Yet LPs rarely manage to impose such terms: Lerner and Nanda (2020) document that LPs who push too hard for transparency risk being shut out of top-performing funds—CalPERS, after leading a pension-fund coalition demanding fee reform in the mid-1990s, was subsequently shunned by leading VC and buyout firms. This asymmetry in bargaining power reflects the market power of the most informed, best-credentialed intermediaries.

This paper asks: when the leader in a financing round holds private information about deal quality, why does it prefer to hide this information until capital is committed, and what are the welfare consequences when followers demand early disclosure? The answer has direct implications for the design of both private VC contracts and public innovation programs (SBIR, Bpifrance) that seek to replicate the certification role of top-tier intermediaries.

This paper. We build a model in which an *informed principal*—the "agency"—knows the exact success probability π of each deal it sources, while outside investors only know the distribution of π. The agency proposes a mechanism specifying how much it co-invests (σ) and what equity share it retains (ρ), then investors decide whether to commit the remaining capital. Two contractual regimes are compared. Under a *blind-pool contract*, the agency proposes a menu before revealing π and investors participate based on expected returns. Under a *deal-by-deal contract*, the agency reveals π upfront and investors can condition their participation on each project's quality.

Main results. We establish four results. First, under deal-by-deal contracts, the IC constraint combined with the pointwise IR constraint forces the optimal allocation to be uniquely uniform—a consequence, not an assumption—but the uniform share generates a strictly positive investor rent on every financed project *above the marginal type*, which the agency cannot recapture. Second, to limit this surplus transfer to investors, the agency sets a financing threshold $\pi^*_\mu > I$, excluding some positive-NPV projects and creating a social inefficiency. Third, under blind-pool contracts, a uniform contract (ρ and σ constant across financed projects) is an optimal solution and is socially optimal: the agency finances all positive-NPV projects within budget, fully extracting the investors' *expected* surplus thanks to the competitive investor market. The contract form is not uniquely determined, but the financing threshold and welfare level are. Fourth, the agency strictly prefers blind-pool over deal-by-deal contracts: its payoff is strictly higher under blind-pool (full expected surplus extraction vs. surplus transfer to investors), and social welfare is simultaneously higher (more positive-NPV projects financed). The two criteria point in the same direction, which explains why blind-pool contracts predominate despite limited partners' preference for deal-by-deal oversight.

Related literature. Our paper contributes to three strands of the literature.

Informed-principal theory. Maskin and Tirole (1992) establish that blind-pool mechanisms are optimal for an informed principal, and Myerson (1993)'s Inscrutability Principle allows restriction to direct revelation mechanisms without loss of generality. Holmstrom and Myerson (1983) characterize mechanisms that are uniformly incentive compatible and interim efficient, which helps justify focusing on this class. We embed these foundations in a VC setting with bilateral budget constraints and a competitive investor market, contributing three elements absent from the baseline framework: (i) explicit optimal thresholds and investment levels in closed form; (ii) a competitive investor market that pins down the investors' expected payoff at zero under blind-pool contracts, directly yielding social optimality; and (iii) a systematic welfare comparison of the two contractual regimes, quantifying the surplus transfer to investors and social inefficiency that arise under deal-by-deal contracts. The optimal control approach used in the deal-by-deal case mirrors the quality-differentiation problem of a monopolist facing a continuum of consumer types (Guesnerie and Laffont, 1984; Kamien and Schwartz, 1991).

LP–GP contracting. Axelson et al. (2009) provides a rationale for blind-pool contracts based on incentive alignment; we emphasize instead the informational advantage of the GP and its certification role. Fang et al. (2015) documents that deal-by-deal financing is too costly due to liquidity constraints and competition in the LP market. Lerner and Nanda (2020) provide direct evidence of LP bargaining weakness—LPs who push for more transparency or fee reform risk being excluded from top funds—and document that the VC industry is highly concentrated, with a small number of top-tier firms capturing most returns. This oligopolistic structure is consistent with, and supports, our informational-monopoly assumption.

Certification and public innovation. The certification role of top-tier intermediaries is studied empirically in Hallen et al. (2014, 2020) and theoretically in Casamatta (2003); Bikhchandani et al. (1992). Public innovation programs are analyzed in Lerner (1996); Howell (2017); Bergemann and Hege (2005); Lerner (2020); Köppl-Turyna et al. (2022).

Section 2 presents the general setup, the timing of the three-stage game, and the incentive-compatibility conditions that discipline the agency's reporting. The agency plays the role of the *principal* in the mechanism-design sense: it proposes the mechanism and holds private information. Section 3 discusses the information structure and motivates the two contractual regimes. Section 4 solves the deal-by-deal framework, showing that revealing π at contracting forces a uniform allocation, leaks rent to investors, and generates a social loss. Section 5 solves the blind-pool framework, showing that concealing π until capital calls allows the principal to fully extract the investors' *expected* surplus and achieve the social optimum. Section 6 concludes with policy implications, notably regarding the government's role as an outside investor in programs such as SBIR or Bpifrance. Proofs are gathered in the Appendix.

2 Setup

Suppose there is no time preference and all the agents below are risk-neutral.

Let's introduce entrepreneurs without initial budget. Each entrepreneur carries a project which requires a sunk investment cost $I \in (0,1)$ and yields a stochastic outcome in $\{0,1\}$. 0 means failure and 1 means success. Without I, the project cannot succeed. An entrepreneur must hence collect at least I from outside sponsors, who can split the investment burden. A project is characterized by its probability of success $\pi \in [0,1]$. Note that the probability of success is equal to the expected outcome. Each entrepreneur knows the expected outcome of his project. Let f be the probability density function of π, and let F denote its cumulative distribution function. A project's *net present value* is defined as the difference of its expected outcome and its investment cost. For notational simplicity, the *market net present value*, which corresponds to the outside option's net present value, is taken to be 0. Then, a project is *worth financing* if its net present value is positive. A project is *financed* if it receives the required investment cost.

(A1): $\mathbb{E}(\pi) - I < 0 < 1 - I \quad$ and $\quad f$ is log-concave

The first part of (A1) means that the average project is not worth financing but that the best project is worth financing. The second part—that $\log f$ is concave—is a standard regularity condition satisfied by the uniform, normal, logistic, and exponential distributions, among others. It implies that the hazard rate $f/(1-F)$ is increasing, a property used in the deal-by-deal analysis.

Potential sponsors are investors with an aggregate budget $B_i \geq 0$. The investors know the distribution of π.

(B1): The investor market is fully competitive.

($B1$) means that investors are profit-driven and they strategically underbid one another in order to secure investment opportunities. It captures for example the setting of a non-cooperative investor oligopoly. From now on, as is standard in the litterature, we will represent the competing investors by a price-taking representative investor.

Financing can also be obtained from an agency with an initial budget $B_a \geq 0$. For each project, the agency knows the exact value of its expected outcome. An *investment allocation* σ specifies, for each project π, the agency's contribution to the investment cost I. The investor is expected to complete the remaining. A *share allocation* ρ specifies, for each project π, the outcome $\rho(\pi)\pi$ obtained by the agency. The investor receives the remaining. An agency's *allocation* μ is a pair (σ, ρ).

(A2): $B_a < I(1 - F(I))$

($A2$) means that the agency cannot finance all worth-financing projects alone: it needs outside investors. This is the economically relevant case—even the largest GPs rely on LP capital to deploy their strategies.

We define *Direct Revelation Mechanisms* (we will use the words "contract" and "mechanism" interchangeably) by the specification of the allocation μ and of each project's expected outcome π. By the *Revelation Principle*, the game's equilibrium will correspond to a truthful equilibrium of direct revelation mechanisms - where the agency is incentivized to report truthfully the expected outcome of each project - at the latest when the mechanism is about to be executed[1]. Let's denote $U_a^\pi(\mu(\hat{\pi}))$ and $U_i^\pi(\mu(\hat{\pi}))$ the respective utilities of the agency and the investor. An allocation μ must be incentive-compatible :

$$U_a^\pi(\mu(\pi)) \geq U_a^\pi(\mu(\hat{\pi})) \quad \forall(\pi, \hat{\pi}) \tag{IC_a}$$

The agency is assumed to be profit-maximizing.

The agency's payoff from a project of true quality π under reported quality $\hat{\pi}$ is:

$$U_a^\pi(\mu(\hat{\pi})) = \rho(\hat{\pi})\pi - \sigma(\hat{\pi}) \quad \forall(\pi, \hat{\pi})$$

Then, the agency's incentive compatibility constraint can be simplified as the following lemma claims.

Lemma 1. *The agency's incentive compatibility constraint is equivalent to its first and second order conditions :*

$$(IC_a) \Leftrightarrow \begin{cases} \frac{d\sigma(\pi)}{d\pi} = \pi \frac{d\rho(\pi)}{d\pi} & \qquad (IC_a^{(1)}) \\ \frac{d\rho(\pi)}{d\pi} \geq 0 & \qquad (IC_a^{(2)}) \end{cases}$$

Proof. See Appendix. □

The investment strategies must be budget balanced both for the agency and the investor:

$$B_a \geq \int \sigma(\pi) f(\pi) d\pi \tag{BC_a}$$

$$B_i \geq \int_{\mu > 0} (I - \sigma(\pi)) f(\pi) d\pi \tag{BC_i}$$

The three-stage game will be such that in the first stage, the agency proposes an allocation μ, giving itself the discretion to choose within the set $\{\mu(\pi)\}$ after the contract is signed. In the second stage, the investor accepts or refuses the contract proposed by the agency. If the investor accepts, the agency executes μ in the third stage.

This game can be interpreted as a joint equity entry into a project, involving investors and a leading agency that coordinates the round table. Note that the agency could, in principle, contract directly with the entrepreneur and leave him exactly the share that should be transferred to the investor, since the entrepreneur's participation requires no equity in our model. The key idea is the fact that the agency's allocation trigger the completion of the remaining investment by outside investors. One example could be that the financing of a startup by a top-tier in the venture capital market could motivate less informed investors to try to enter in the capital. Another one could be the relationship

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[1] Maskin and Tirole (1992)

between limited partners and venture capital firms and their contractual framework for entering in the venture capital market.

An allocation is said to be *uniform* when it is constant, except in the no-investment case. An allocation is said to be *socially optimal* if it maximizes the welfare (under budget constraints):

$$\int (\pi - I) f(\pi) d\pi$$

The socially optimal threshold is $\pi^*_\mu = \max((1-F)^{-1}(\frac{B_a+B_i}{I}), I)$: the planner finances all projects above π^*_μ until the budget is exhausted.

(A3): $B_a + B_i \geq I(1 - F(I))$

$(A3)$ means that the total available capital is sufficient to finance all worth-financing projects. Under $(A3)$, the socially optimal threshold reduces to $\pi^*_\mu = I$: every positive-NPV project is financed. The "cash-poor" case $(B_a + B_i < I(1 - F(I)))$ is treated in the Appendix.

3 Information structure

When the investor knows π, the agency only has a first-mover advantage but not any informational edge over the market. This can be thought of as the relationship between accelerators and venture capital firms. In this case, the agency's incentive constraint is absent.

Proposition 1. *When the investor knows π, any socially optimal allocation which obeys to:*

$$\rho^*(\pi) = 1 - \frac{I - \sigma^*(\pi)}{\pi}$$

solves the agency's problem.

Proof. See Appendix. □

Example 1. *Let $\pi \sim U[0,1]$, $I = 0.6$, $B_a = 0.1$, $B_i = 0.2$. Assumptions (A1)–(A3) are satisfied: $\mathbb{E}(\pi) = 0.5 < 0.6 = I < 1$, $B_a = 0.1 < 0.24 = I(1-I)$, and $B_a + B_i = 0.3 \geq 0.24$. Take $\sigma^* = 0.1$ (agency's budget binding). Then $\rho^*(\pi) = 1 - (0.6 - 0.1)/\pi = 1 - 0.5/\pi$ for $\pi \geq I$. At $\pi = 0.6$: $\rho^*(0.6) = 1/6 \approx 0.167$. At $\pi = 1$: $\rho^*(1) = 0.5$. The agency's payoff per project is $U_a(\pi) = \rho^*(\pi)\pi - \sigma^* = \pi - I$, confirming full surplus extraction. The investor earns $U_i(\pi) = 0$ on every project.*

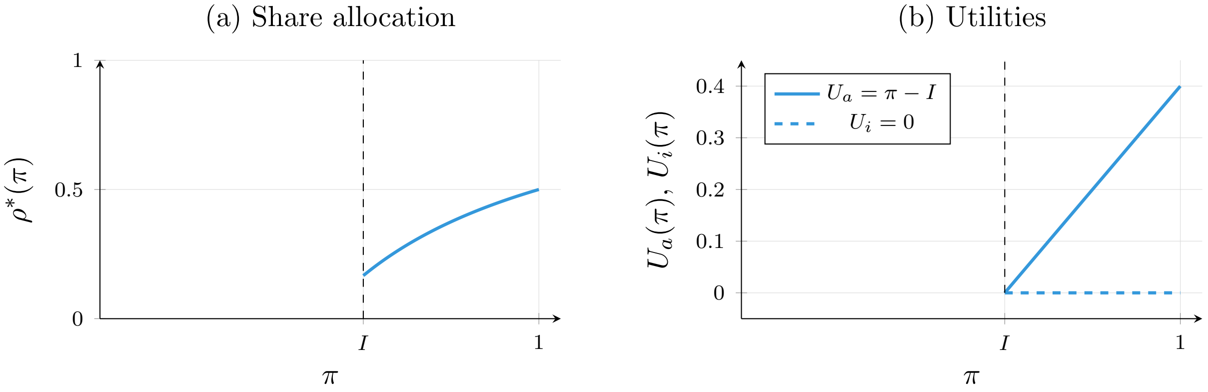

Figure 1: Informed investors. $\pi \sim U[0,1]$, $I = 0.6$, $B_a = 0.1$, $B_i = 0.2$, $\sigma^* = 0.1$. (a) The agency tailors $\rho^*(\pi) = 1 - (I - \sigma^*)/\pi$, increasing in π. (b) The agency captures the full surplus $\pi - I$ on each project; the investor earns zero rent.

Indeed, since the agency is profit-maximizing, the investor's rationality constraint is binding for each project. Otherwise, since the investor market is fully competitive, the agency could contract with a cheaper investor, thereby making more profit. Thus, the agency's optimization problem directly reduces to a welfare maximization problem under budget constraints. This proposition means that when there is no asymmetric information, the agency extracts all the surplus from the investor project-wise, thanks to its first mover advantage. This can be interpreted as a rationale for seed venture capital activities: the first investor in a good startup can extract the surplus of subsequent investors, even if the latter can perform a qualitative due diligence.

From now on, assume there is an information asymmetry:

$(B2)$ The investor does not know π.

$(B2)$ raises the question of the stage when the agency reports π to the investor. The intuition is that the agency should reveal π as late as possible—after investors have committed capital—which is indeed the optimal mechanism[2]. This corresponds to the *blind-pool* structure widely used in VC: limited partners commit capital without knowing which entrepreneurs will be financed. Another example is when a top-tier firm's investment in a startup triggers less informed investors to co-invest: the leader provides just enough information for followers to participate. There would be a loss for the agency if π was revealed at the contracting stage. Investors are more likely to push towards *deal-by-deal* contracts, as it is more advantageous to them. Yet, as Lerner and Nanda (2020) document, LPs who actively lobby for more transparency risk being excluded from the best-performing funds—a concrete illustration of the informational and market-power asymmetry that underpins our model. The following sections provide a rationale for why blind-pool contracts naturally arise in the relationship between venture capital firms and limited partners, and why deal-by-deal contractual frameworks are not socially optimal.

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[2] Maskin and Tirole (1992)

4 Deal-by-deal contracts

Deal-by-deal contracts translate into contracts where the agency reveals π at the contracting stage:

$$\forall \pi, (1-\rho(\pi))\pi - (I - \sigma(\pi)) \geq 0 \qquad (IR_{i,p})$$

Lemma 2. *No project with $\pi < I$ is financed in any optimal mechanism.*

Proof. If $\pi < I$, then $\pi - I < 0$: the project has negative expected net value. The investor's payoff from financing it is $(1-\rho(\pi))\pi - (I - \sigma(\pi)) \leq \pi - I < 0$ whenever $\sigma(\pi) \leq \rho(\pi)\pi$ (i.e. the agency does not subsidize the investor beyond the project's return). Thus $(IR_{i,p})$ cannot be satisfied unless the agency transfers more than the project generates, which would also make the agency's payoff negative. Since both parties lose, no rational mechanism finances projects below I. $\square$

This lemma establishes that a financing threshold $\pi_\mu \geq I$ arises endogenously from the parties' rationality, not from an exogenous assumption.

Proposition 2. *In a deal-by-deal contractual framework, the agency's optimal allocation is uniform.*

Proof. See Appendix. $\square$

This proposition is counter-intuitive especially because the agency does not take any advantage from its revealing constraint, thereby, as we will see later, giving up a strictly positive rent to the investor on all the projects until the worst financed project. The following lemma shows that strictly increasing allocations are also suboptimal:

Lemma 3. *There is no interval on which the optimal share allocation is strictly increasing and the investor participation constraint is binding.*

Proof. Suppose there is an interval on which $(1-\rho(\pi))\pi - (I - \sigma(\pi)) = 0$ and $\rho'(\pi) > 0$. Differentiating the IR with respect to π: $-\rho'(\pi)\pi + (1-\rho(\pi)) + \sigma'(\pi) = 0$. Substituting $\sigma'(\pi) = \pi\rho'(\pi)$ from $(IC_a^{(1)})$ yields $1 - \rho(\pi) = 0$, contradicting $\rho'(\pi) > 0$. $\square$

The economic intuition is the following. When investors demand to observe π before committing capital, they also implicitly require a uniform contract: any contract that conditions terms on π would give the agency an incentive to overstate project quality in order to obtain more favorable terms. A uniform contract neutralizes this incentive, since the agency's share and co-investment are the same regardless of the announced π. Lemma 3 makes this precise: if one tries to increase the agency's share on better projects while keeping the investor's participation constraint binding, the IC forces $\rho = 1$—the agency would have to finance the entire project alone, making the contract vacuous. Proposition 2 establishes the result formally via optimal control, ruling out $\rho'(\pi) > 0$ on any interval. Together, they show that uniformity is a *consequence* of the IC constraint interacting with a pointwise IR, not an assumption. The uniform share is then pinned down by the binding IR at the marginal project π_μ^*: the agency sets ρ^* so that the worst financed project exactly breaks even for the investor. All higher-quality projects yield a strictly positive investor rent—the cost of deal-by-deal revelation.

Thus, the agency solves:

$$\max_{(\pi_\mu,\sigma,\rho)} \left(\int_{\pi_\mu}^{1} (\rho\pi - \sigma) f(\pi) d\pi \right)$$

under:

$$\forall \pi \geq \pi_\mu, \quad (1-\rho)\pi - (I-\sigma) \geq 0 \Leftrightarrow (1-\rho)\pi_\mu \geq I - \sigma \tag{$IR_{i,u}$}$$

Proposition 3. *In a deal-by-deal contractual framework, any optimal allocation satisfies:*

1. *Some worth-financing projects are not financed:* $\pi_\mu^* > I$.
2. *The agency saturates its budget:*

$$\sigma^* = \frac{B_a}{\int_{\pi_\mu^*}^{1} f(\pi) d\pi}$$

3. *The share allocation is pinned down by the threshold:*

$$\rho^* = 1 - \frac{I - \sigma^*}{\pi_\mu^*}$$

4. *The optimal threshold* π_μ^* *satisfies the first-order condition:*

$$(\pi_\mu^* - I) f(\pi_\mu^*) = \frac{\int_{\pi_\mu^*}^{1} (\pi - \pi_\mu^*) f(\pi)\, d\pi}{\pi_\mu^* \int_{\pi_\mu^*}^{1} f(\pi)\, d\pi} \cdot \sigma^* \cdot f(\pi_\mu^*) + \frac{I - \sigma^*}{(\pi_\mu^*)^2} \int_{\pi_\mu^*}^{1} \pi f(\pi)\, d\pi$$

   The right-hand side is strictly positive, which forces $\pi_\mu^* > I$*: the agency excludes some positive-NPV projects to limit the surplus transferred to investors.*

Example 2. *Same parameters as Example 1. For* $f \equiv 1$ *on* $[0,1]$*, the first-order condition (item 4) reduces to* $2(\pi_\mu^*)^3 - I(\pi_\mu^*)^2 - I + B_a = 0$*, i.e.* $2(\pi_\mu^*)^3 - 0.6(\pi_\mu^*)^2 - 0.5 = 0$*. This cubic has a unique root in* $(I, 1)$*:* $\pi_\mu^* \approx 0.748$ *(uniqueness is proved in the Appendix). Then:*

- $\pi_\mu^* \approx 0.748 > 0.6 = I$. *Projects with* $\pi \in [0.6, 0.748)$ *are positive-NPV but excluded.*
- $\sigma^* = B_a/(1 - \pi_\mu^*) = 0.1/0.252 \approx 0.397$.
- $\rho^* = 1 - (I - \sigma^*)/\pi_\mu^* = 1 - 0.203/0.748 \approx 0.729$.

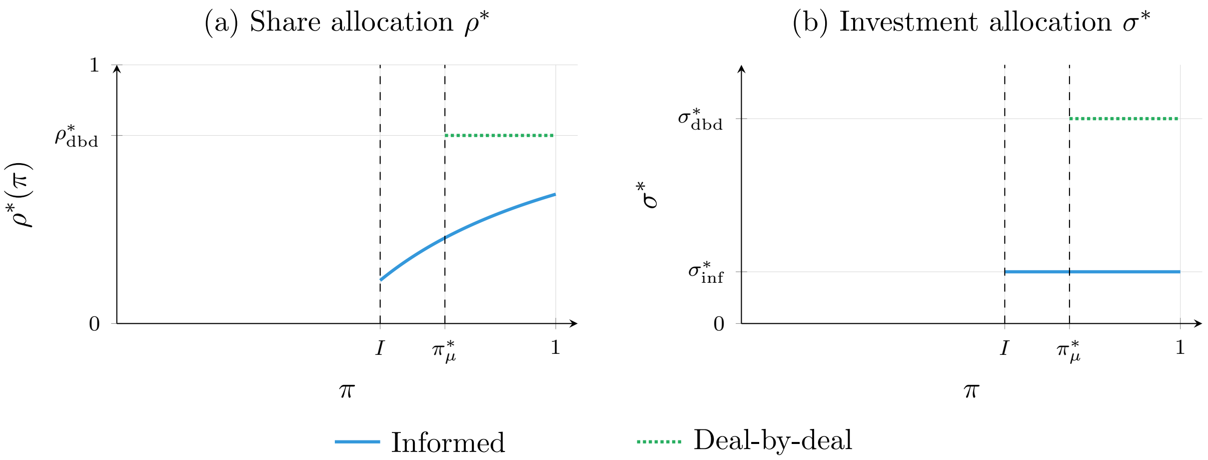


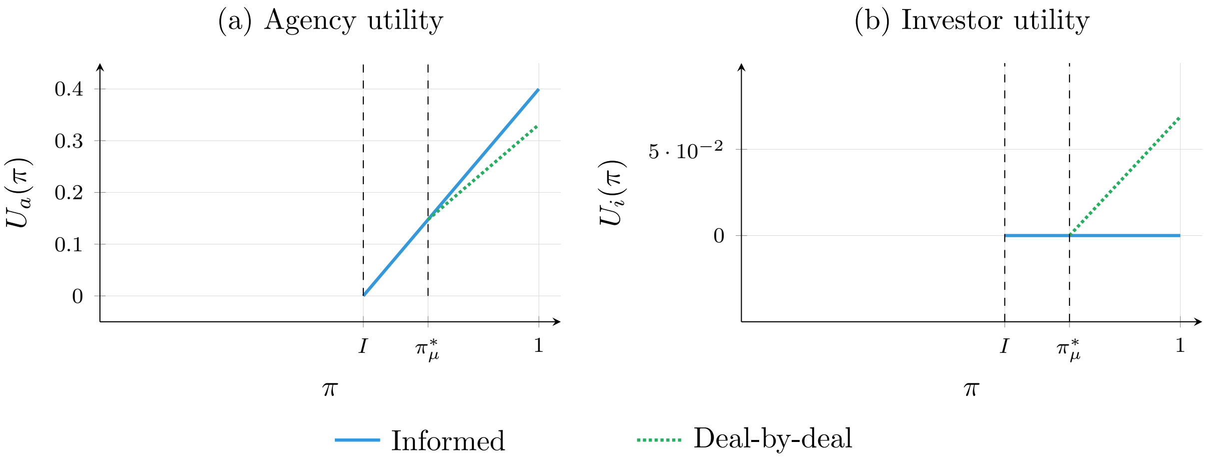
Figure 2: Informed investors vs. deal-by-deal. Parameters: $\pi \sim U[0,1]$, $I = 0.6$, $B_a = 0.1$, $B_i = 0.2$. Under informed investors: $\pi^*_\mu = I$, $\rho^*(\pi) = 1 - (I - \sigma^*)/\pi$, $\sigma^* = 0.1$. Under deal-by-deal: $\pi^*_\mu \approx 0.748 > I$, $\rho^* \approx 0.727$ (uniform), $\sigma^* \approx 0.396$. The deal-by-deal agency finances fewer projects but invests more per project.

The agency's payoff per project is $U_a(\pi) = 0.729\pi - 0.397$*. The investor's payoff is* $U_i(\pi) = 0.271\pi - 0.203$*, which equals zero at* $\pi = \pi^*_\mu$ *and is strictly positive for* $\pi > \pi^*_\mu$*: at* $\pi = 1$*,* $U_i(1) \approx 0.068$*. Compared to Example 1, the agency loses surplus on two fronts: a narrower range of financed projects and a positive investor rent on each financed project.*

The logical chain behind Proposition 3 mirrors Proposition 4, but with a crucial difference at each step. **(1) Pointwise IR.** Because investors observe π before committing, each project must individually satisfy ($IR_{i,p}$): $(1-\rho)\pi \geq I - \sigma$ for all $\pi \geq \pi^*_\mu$. **(2) Uniform contract.** Combined with the IC constraint, this pointwise IR forces the optimal allocation to be uniform (Lemma 3 and Proposition 2): the agency cannot tailor ρ project-by-project without violating either IC or IR. **(3) Surplus transfer.** Because the uniform share ρ^* is set so that the *marginal* project π^*_μ just breaks even for the investor, all projects with $\pi > \pi^*_\mu$ yield a strictly positive investor rent $(1-\rho^*)\pi - (I - \sigma^*) > 0$. This rent cannot be extracted by the agency. **(4) Inefficiency.** To limit these surplus transfers, the agency sets $\pi^*_\mu > I$: it voluntarily excludes some positive-NPV projects (those with $\pi \in [I, \pi^*_\mu)$) rather than finance them at the cost of transferring surplus on all financed projects. The optimal threshold $\pi^*_\mu > I$ balances these two forces.

In deal-by-deal frameworks, investors obtain a strictly positive rent on every financed project above the marginal type. This explains why limited partners increasingly push toward deal-by-deal contracts with venture capital firms. However, such contracts are not socially desirable—the exclusion of positive-NPV projects represents a social inefficiency—and they are unlikely to emerge naturally given the informational advantage of top-tier GPs and the competitive structure of the LP market.

Remark (why deal-by-deal $\neq$ informed investors). Under informed investors, the investor knows π independently of the agency: there is no IC constraint, and the agency can tailor $\rho(\pi)$ to each project, extracting the full surplus. Under deal-by-deal, the investor learns π *because the agency reports it.* If the contract terms depended on the announced π—for instance, a higher ρ for better projects—the agency would have an incentive to overstate project quality to obtain more favorable terms. The IC constraint prevents this, and combined with the pointwise IR, forces the contract to be uniform (Proposition 2, Lemma 3). Project-by-project extraction is therefore impossible as long as the investor relies on the agency's report. It would require the investor to observe π with noise independently, relaxing the pointwise IR to an interim condition—an extension left to future work.


(a) Agency utility

(b) Investor utility

Figure 3: Utilities per project: informed vs. deal-by-deal. Same parameters as Figure 2. (a) Agency utility: informed captures $\pi - I$, deal-by-deal captures less on a narrower range. (b) Investor utility: zero under informed investors, strictly positive for $\pi \geq \pi^*_\mu$ under deal-by-deal.

5 Blind-pool contracts

Blind-pool contracts translate into the following investor participation constraint[3]:

$$\int_{\mu>0} \big[(1-\rho(\pi))\pi - (I - \sigma(\pi))\big] f(\pi)\, d\pi \geq 0 \tag{IR_i}$$

where $\{\mu > 0\}$ denotes the set of projects that receive financing. (IR_i) means that the agency presents to investors a menu $(\rho(\pi), \sigma(\pi))$ and investors commit capital if they break even on average across the financed pool. They must also ensure that the agency has no interest in deviating from the proposed contract or in presenting one project as another. Maskin and Tirole (1992)'s first theorem shows that (IR_i) and (IC_a) satisfy these conditions. The revelation at the third stage can be thought of as the moment

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[3] Maskin and Tirole (1992)

of capital calls when GPs attribute to a given project its valuation and ask LPs the corresponding investment amount. Once (IR_i) binds, the agency's objective reduces to $\int_{\mu>0}(\pi - I)f(\pi)\,d\pi$, independent of the specific form of $(\rho(\pi), \sigma(\pi))$: any allocation satisfying IC and the binding (IR_i) is optimal. In particular, a uniform allocation (ρ^*, σ^*) with a financing threshold π_μ achieves the optimum. The following proposition characterizes it:

Proposition 4. *In a blind-pool contractual framework, a uniform allocation* (ρ^*, σ^*) *with financing threshold* π_μ *is an optimal solution, characterized by:*

1. *The financing threshold is* $\pi_\mu^* = I$*: all positive-NPV projects are financed.*
2. *The agency's share is:*
$$\rho^* = 1 - \frac{(1 - F(I))(I - \sigma^*)}{\int_I^1 \pi f(\pi)\,d\pi}$$
3. *The agency's investment share belongs to:*
$$\sigma^* \in \left[I - \frac{B_i}{1 - F(I)},\ \frac{B_a}{1 - F(I)}\right] \cap (0, I]$$

The contract form is not uniquely determined—any allocation satisfying IC and the binding (IR_i) *is optimal—but the welfare level is. The uniform allocation with threshold* I *is one such solution. The cash-poor case* ($B_a + B_i < I(1 - F(I))$) *is treated in the Appendix.*

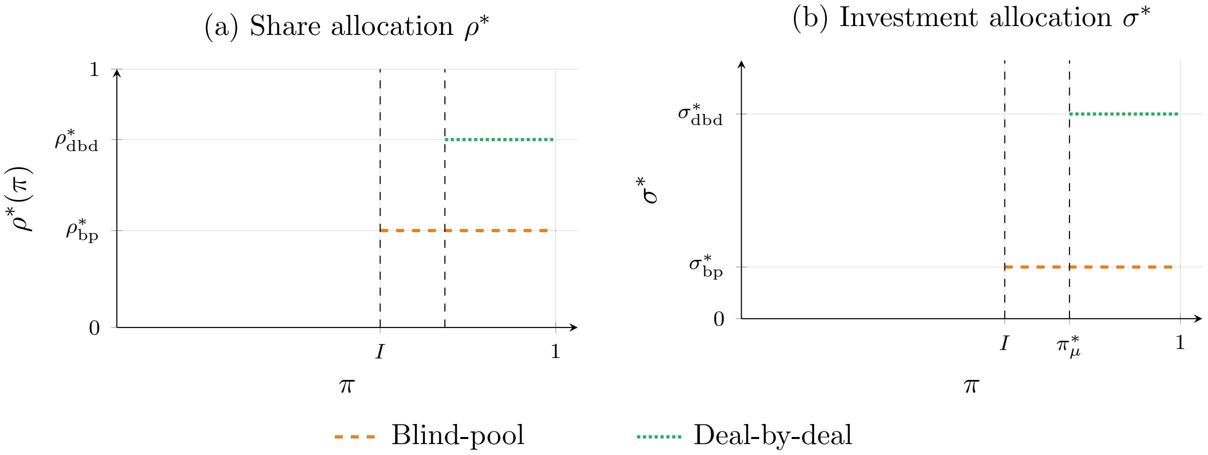
Proof. See Appendix. □

Example 3. *Same parameters as Examples 1–2. The financing threshold is* $\pi_\mu^* = I = 0.6$*: all positive-NPV projects are financed.*

- $\rho^* = 1 - (1 - F(I))(I - \sigma^*)/\int_I^1 \pi\,d\pi = 1 - 0.4 \times 0.5\,/\,0.32 = 0.375$.
- $\sigma^* \in [I - B_i/(1 - I),\ B_a/(1 - I)] \cap (0, I] = [0.1,\ 0.25]$. *Take* $\sigma^* = 0.1$ *(agency's budget binding).*

The agency's payoff per project is $U_a(\pi) = 0.375\pi - 0.1$*. The investor's payoff is* $U_i(\pi) = 0.625\pi - 0.5$*: negative for* $\pi < 0.8$*, positive for* $\pi > 0.8$*, but zero* on average *over* $[I, 1]$*. Compared to Example 2, the blind-pool contract finances more projects (*$\pi_\mu^* = 0.6$ *vs.* 0.748*) and transfers no surplus to investors in expectation. The agency strictly prefers the blind-pool.*

The logical chain behind Proposition 4 runs as follows. **(1) IR in expectation.** Because the investor does not observe π before committing, the participation constraint (IR_i) is only required to hold in expectation. **(2) Competitive market ⇒ full extraction.** The fully competitive investor market implies that the binding IR in expectation allows the agency to extract all investor surplus on average: any slack in (IR_i) would be competed away. Substituting the binding IR into the agency's objective collapses it to a pure


(a) Share allocation ρ^*

(b) Investment allocation σ^*

Figure 4: Blind-pool vs. deal-by-deal. Same parameters as Figure 2. Blind-pool: $\pi^*_\mu = I$, $\rho^* = 0.375$, $\sigma^* = 0.1$. Deal-by-deal: $\pi^*_\mu \approx 0.748$, $\rho^* \approx 0.727$, $\sigma^* \approx 0.396$. The blind-pool contract finances all positive-NPV projects; the deal-by-deal contract excludes those with $\pi \in [I, \pi^*_\mu)$.

welfare-maximization program $\max \int_{\pi_\mu}^{1} (\pi - I) f(\pi)\, d\pi$. **(3) Social optimum.** This objective is identical to the social planner's, so the agency's blind-pool strategy is socially optimal: it finances the maximum number of worth-financing projects within the available budget. Budget constraints then pin down the financing threshold and investment split in Proposition 4.

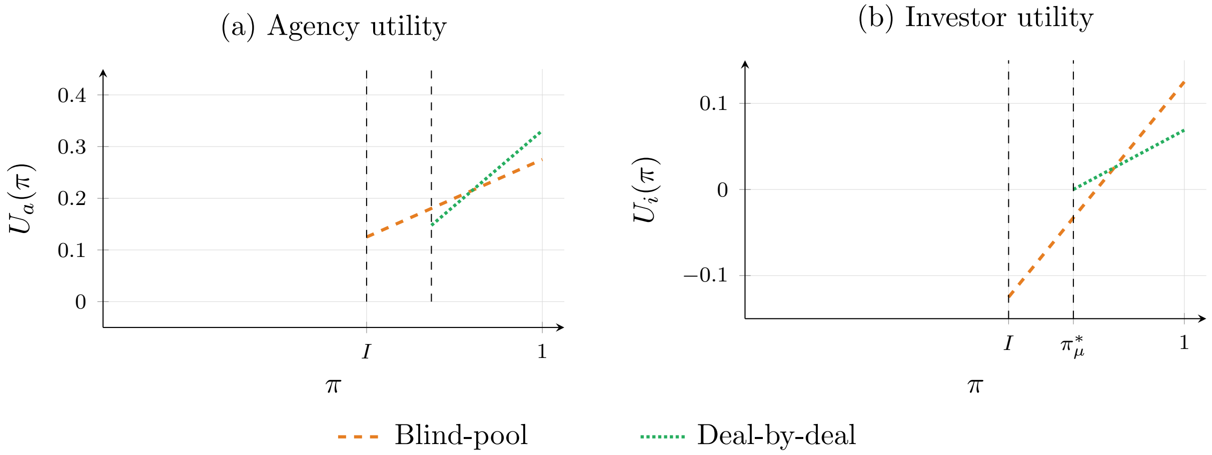


Figure 5: Utilities: blind-pool vs. deal-by-deal. Same parameters. (a) Agency utility: blind-pool yields higher expected profit (wider range, starts at I). (b) Investor utility: zero *on average* under blind-pool (negative for low-quality, positive for high-quality projects), strictly positive $\forall \pi \geq \pi_\mu^*$ under deal-by-deal. The investor prefers deal-by-deal; the agency prefers blind-pool.

Summary: Comparison of Contracting Mechanisms

Table 1: Key features of the three contractual frameworks

	Informed investors	**Blind-pool**	**Deal-by-deal**
IR constraint	Pointwise $\forall\pi$	In expectation	Pointwise $\forall\pi$
IC constraint	Absent	Present	Present
Optimal allocation	Non-uniform: $\rho^*(\pi) = 1 - \frac{I-\sigma^*(\pi)}{\pi}$	Uniform ρ^*	Uniform ρ^*
Investment	Flexible $\sigma^*(\pi)$	$\sigma^* \in (0, I]$	$\frac{B_a}{\int_{\pi_\mu^*}^1 f\,d\pi}$
Threshold π_μ^*	I	I	$\pi_\mu^* > I$
Investor rent	0	0	> 0 on $[\pi_\mu^*, 1]$
Socially optimal	Yes	Yes	No

6 Conclusion

Our results rest on assumptions that deserve acknowledgment. First, the *informational monopoly* condition—that the agency knows π exactly and is the unique informed party—is central to tractability and is stronger than what is observed in practice. However, the VC industry is highly concentrated: Lerner and Nanda (2020) document that a small number of top-tier firms capture most of the sector's returns, suggesting that an oligopolistic structure—rather than perfect competition among informed intermediaries—is the empirically relevant benchmark. In this sense, the monopoly assumption, while stylized, captures the right qualitative feature of the market, and relaxing it to a small-numbers oligopoly would not fundamentally alter the main insights. Second, we assume *no externalities across projects*; in reality, financing one startup may crowd out capital for another, or generate positive network spillovers. Third, the investment cost I is *identical across projects*; in practice, early-stage and late-stage deals require very different capital amounts, and allowing $I(\pi)$ to vary could affect the optimal allocation. Fourth, the agency's compensation is entirely captured by (ρ, σ); we abstract from *management fees*, which in practice depend on the capital deployed and could create an incentive to inflate investment costs.

Our model sheds light on certification contractual frameworks in venture capital. The relationship between accelerators and venture capital firms corresponds to the informed-investors benchmark: the first investor in a good startup can extract the surplus of subsequent investors thanks to its first-mover advantage. More generally, when the leader agency has the capacity to identify good projects and is recognized as such by outside investors, its optimal strategy is to finance all positive-NPV projects within the available budget while concealing private information until execution. This strategy is socially optimal. The problem is that in a constrained economy, governments may wish to finance projects that are not the best by market standards but have high social value—national priorities in energy, defense, or environmental sustainability. Our model clarifies when public intervention is warranted. *If the government has genuine informational expertise* (like a top-tier accelerator), our model predicts it should behave like the agency: finance as many positive-NPV projects as possible while concealing private valuations from co-investors. However, the empirical record of public agencies as expert screeners is mixed. Howell (2017) studies the SBIR program and finds that while SBIR grants increase the probability of subsequent VC financing, the certification channel—whereby the public grant signals quality to private investors—is not identified as the main mechanism; rather, the effect operates through a financing channel—grants relax cash constraints and enable prototyping, which in turn attracts VC interest. This is consistent with the view that public programs operate primarily as budget providers rather than as certification agents (see also Lerner, 2020; Köppl-Turyna et al., 2022). *If the government lacks informational advantage*—the more realistic case—it should not attempt to replicate the agency's screening role. Instead, it can act as an outside investor (a fund-of-funds or a co-investor in VC deals), committing capital through a blind-pool structure and relying on the informational monopolist to allocate it optimally. In this role, the government breaks even on average and does not distort the private market's certification mechanism. For example, Bpifrance is best interpreted not as an expert screener but as a budget-provider whose capital is allocated by informed private intermediaries. Recent evidence supports this interpretation. Following a co-investment principle, Bpifrance complements private supply without substituting for it: on average, it represented only 16% of capital raised in rounds financed by its direct funds. Through its fund-of-funds activity, Bpifrance's €5.9 billion committed over ten years enabled 180 French VC partner funds to raise €32 billion in private capital—a leverage ratio of more than 5:1. Moreover, 78% of fund managers and subscribers surveyed confirm that Bpifrance's public investment helped attract additional private co-investors (Gazaniol, 2025). More broadly, our model provides a rationale for the predominance of blind-pool contracts in GP–LP contracting. The GP acts as the informational monopolist and exploits its informational advantage by concealing private valuations until LPs have committed capital. Lerner and Nanda (2020) document that this pattern is not merely contractual convenience: LPs who demand greater transparency or push for deal-by-deal approval risk being excluded from top-performing funds, confirming that GPs' market power is the binding constraint. The VC industry is highly concentrated, with a small number of top-tier firms capturing most of the sector's returns—an oligopolistic structure in which each leading fund behaves approximately as an informational monopolist vis-à-vis its LP base. Our model formalizes this insight: the blind-pool structure is not just customary but *optimal* for an informed principal facing competitive uninformed investors. Moreover, blind-pool dominance is not only privately optimal for the GP—it is also socially desirable. A shift toward discretionary deal-by-deal arrangements—for instance through annual advisory committees reviewing each investment opportunity—would force the GP to reveal project quality upfront, triggering the surplus-transfer mechanism analyzed in Section 4. To limit the resulting investor rents, the GP would raise the financing threshold above I, excluding positive-NPV projects that would have been funded under a blind-pool structure. The social cost is a social inefficiency: viable projects go unfinanced, not because they lack merit, but because the contractual framework makes it too costly for the GP to bring them to market.

7 Appendix

7.1 Proof of Lemma 1 (incentive compatibility).

$\boxed{\Rightarrow}$

The first-order condition is:

$$\frac{\partial V}{\partial \hat{\pi}}(\pi, \pi) = 0 \Leftrightarrow \frac{d\sigma(\pi)}{d\pi} = \pi \frac{d\rho(\pi)}{d\pi}$$

Differentiating:

$$\frac{d^2\sigma(\pi)}{d\pi^2} = \frac{d\rho(\pi)}{d\pi} + \pi \frac{d^2\rho(\pi)}{d\pi^2}$$

and the second-order condition is:

$$\frac{d^2\sigma(\pi)}{d\pi^2} \geq \pi \frac{d^2\rho(\pi)}{d\pi^2}$$

which by substitution yields:

$$\frac{d\rho(\pi)}{d\pi} \geq 0$$

$\boxed{\Leftarrow}$

Consider:

$$\frac{\partial V}{\partial \hat{\pi}}(\pi, \hat{\pi}) = \Big(\frac{\partial(\pi \rho(\hat{\pi}))}{\partial \rho} \frac{d\rho(\hat{\pi})}{d\pi} - \frac{d\sigma(\hat{\pi})}{d\pi} \Big)$$

By writing the first-order condition in $\hat{\pi}$, we obtain:

$$\frac{\partial(\hat{\pi} \rho(\hat{\pi}))}{\partial \rho} \frac{d\rho(\hat{\pi})}{d\pi} = \frac{d\sigma(\hat{\pi})}{d\pi}$$

thus,

$$\frac{\partial V(\pi, \hat{\pi})}{\partial \hat{\pi}} = \Big(\frac{\partial(\pi \rho(\hat{\pi}))}{\partial \rho} - \frac{\partial(\hat{\pi} \rho(\hat{\pi}))}{\partial \rho} \Big) \frac{d\rho(\hat{\pi})}{d\pi}$$

but by the *Mean Value Theorem*, the sign of the right-hand side is that of:

$$\frac{\partial^2(\pi^* \rho(\hat{\pi}))}{\partial \rho \partial \pi} (\pi - \hat{\pi}) \frac{d\rho(\hat{\pi})}{d\pi}$$

for some π^* between π and $\hat{\pi}$. Since $\frac{\partial^2(\pi^* \rho(\hat{\pi}))}{\partial \rho \partial \pi} = 1$, combined with the second-order condition we obtain that $\hat{\pi} \mapsto V(\pi, \hat{\pi})$ is increasing until $\hat{\pi} = \pi$ and then decreasing, which concludes the proof.

7.2 Proof of Proposition 1 (informed investors).

Step 1: The investor's rationality constraint binds

Since the agency is profit-maximizing, the investor's rationality constraint is binding for each project. Otherwise, since the investor market is fully competitive, the agency could contract with a cheaper investor, thereby making more profit:

$$1-\rho^*(\pi) > \frac{I-\sigma^*(\pi)}{\pi} \Rightarrow \exists \rho^{\#}(\pi) \in \Big(\rho^*(\pi), 1-\frac{I-\sigma^*(\pi)}{\pi}\Big),$$
$$\int_{\pi^*_\mu}^{1} (\rho^*(\pi)\pi - \sigma^*(\pi)) f(\pi)\, d\pi < \int_{\pi^*_\mu}^{1} (\rho^{\#}(\pi)\pi - \sigma^*(\pi)) f(\pi)\, d\pi$$

and $(\sigma^*, \rho^{\#})$ satisfies (BC_a), (BC_i), and (IR_i), a contradiction. Thus:

$$\rho^*(\pi) = 1 - \frac{I-\sigma^*(\pi)}{\pi}$$

Step 2: Reducing the agency's optimization problem

The agency's optimization problem reduces to:

$$\max_{(\pi_\mu,\sigma,\rho)} \int_{\pi_\mu}^{1} \Big(\Big(1-\frac{I-\sigma(\pi)}{\pi}\Big)\pi - \sigma(\pi)\Big) f(\pi)\, d\pi$$
$$= \max_{(\pi_\mu,\sigma,\rho)} \int_{\pi_\mu}^{1} (\pi - I) f(\pi)\, d\pi$$

and:

$$\rho(\pi) = 1 - \frac{I-\sigma(\pi)}{\pi}$$

7.3 Proof of Proposition 2 (uniformity under deal-by-deal).

The agency's program can be rewritten as the following optimal control problem:

$$\max_{(\pi_\mu,\sigma,\rho)} \int_{\pi_\mu}^{1} (\rho(\pi)\pi - \sigma(\pi)) f(\pi)\, d\pi$$

under:

$$B_a - \int_{\pi_\mu}^{1} \sigma(\pi) f(\pi)\, d\pi \geq 0 \qquad (BC_a)$$
$$B_i - \int_{\pi_\mu}^{1} (I-\sigma(\pi)) f(\pi)\, d\pi \geq 0 \qquad (BC_i)$$
$$(1-\rho(\pi))\pi - (I-\sigma(\pi)) \geq 0 \qquad (IR_{i,p})$$
$$\rho'(\pi) = u(\pi), \quad \sigma'(\pi) = \pi u(\pi) \qquad (IC_a^{(1)})$$
$$u(\pi) \geq 0 \qquad (IC_a^{(2)})$$
$$\sigma(\pi_\mu), \rho(\pi_\mu), \sigma(1), \rho(1) \text{ free} \qquad (LC)$$

Step 1: Applying Pontryagin's Maximum Principle

The Hamiltonian is:

$$\begin{aligned} H(\pi) =&(\rho(\pi)\pi - (1 + (\lambda_a - \lambda_i))\sigma(\pi))f(\pi) \\ &+\lambda_1(\pi)u(\pi) + \lambda_2(\pi)\pi u(\pi) \\ &+\eta_1(\pi)u(\pi) + \eta_2(\pi)(\pi - I - \rho(\pi)\pi + \sigma(\pi)) \end{aligned}$$

- Optimality condition

$$\frac{\partial H}{\partial u} = 0 \Leftrightarrow \lambda_1(\pi) + \lambda_2(\pi)\pi + \eta_1(\pi) = 0$$

- Multiplier conditions

$$\begin{aligned} \lambda_1'(\pi) &= -\frac{\partial H}{\partial \rho} = \eta_2(\pi)\pi - \pi f(\pi) \\ \lambda_2'(\pi) &= -\frac{\partial H}{\partial \sigma} = (1 + (\lambda_a - \lambda_i))f(\pi) - \eta_2(\pi) \end{aligned}$$

- Complementarity slackness conditions

$$\begin{aligned} &\lambda_a \geq 0, \lambda_i \geq 0, \eta_1(\pi) \geq 0, \eta_2(\pi) \geq 0 \\ &\lambda_a\Big(B_a - \int_{\pi_\mu}^{1} \sigma(\pi)f(\pi)\,d\pi\Big) = 0 \\ &\lambda_i\Big(B_i - \int_{\pi_\mu}^{1} (I - \sigma(\pi))f(\pi)\,d\pi\Big) = 0 \\ &\eta_1(\pi)u(\pi) = 0 \\ &\eta_2(\pi)(\pi - I - \rho(\pi)\pi + \sigma(\pi)) = 0 \end{aligned}$$

- Transversality conditions

$$\lambda_1(\pi_\mu) = 0, \lambda_2(\pi_\mu) = 0, \lambda_1(1) = 0, \lambda_2(1) = 0$$

Step 2: Rearranging the optimality condition and establishing $\lambda_a > \lambda_i$

From the optimality condition on σ in the Hamiltonian, maximizing H with respect to the allocation between σ and $I-\sigma$ yields the first-order condition $\lambda_a - \lambda_i = \frac{\int_{\pi_\mu}^{1}(\pi - \pi_\mu)f(\pi)\,d\pi}{\pi_\mu \int_{\pi_\mu}^{1} f(\pi)\,d\pi} > 0$. The inequality holds because $\pi > \pi_\mu$ on the integration domain and under $(A2)$ the agency cannot self-finance. Hence $\lambda_a > \lambda_i$.

Now, from the optimality and transversality conditions:

$$\begin{aligned}-\eta_1(\pi) &= \lambda_1(\pi) + \lambda_2(\pi)\pi \\ &= \int_{\pi_\mu}^{\pi} \lambda_1'(v)\,dv + \Big(\int_{\pi_\mu}^{\pi} \lambda_2'(v)\,dv\Big)\pi \\ &= \int_{\pi_\mu}^{\pi} \big(\eta_2(v)v - vf(v) + \pi((1+(\lambda_a-\lambda_i))f(v) - \eta_2(v))\big)\,dv \\ &= \int_{\pi_\mu}^{\pi} (\eta_2(v)v - \pi\eta_2(v))\,dv - \int_{\pi_\mu}^{\pi} vf(v)\,dv + (1+(\lambda_a-\lambda_i))\pi(F(\pi)-F(\pi_\mu))\end{aligned}$$

By the *Fundamental Theorem of Calculus*:

$$\begin{aligned}\pi(F(\pi)-F(\pi_\mu)) &= \int_{\pi_\mu}^{\pi} \frac{d(v(F(v)-F(\pi_\mu)))}{dv}\,dv \\ &= \int_{\pi_\mu}^{\pi} (vf(v) + (F(v)-F(\pi_\mu)))\,dv\end{aligned}$$

Thus:

$$\begin{aligned}-\eta_1(\pi) &= \int_{\pi_\mu}^{\pi} \eta_2(v)(v-\pi)\,dv - \int_{\pi_\mu}^{\pi} vf(v)\,dv + (1+(\lambda_a-\lambda_i))\int_{\pi_\mu}^{\pi} (vf(v) + (F(v)-F(\pi_\mu)))\,dv \\ &= \int_{\pi_\mu}^{\pi} \eta_2(v)(v-\pi)\,dv + (1+(\lambda_a-\lambda_i))\int_{\pi_\mu}^{\pi} (F(v)-F(\pi_\mu))\,dv + (\lambda_a-\lambda_i)\int_{\pi_\mu}^{\pi} vf(v)\,dv\end{aligned}$$

Step 3: Showing $u = 0$

Suppose there exists an interval $\mathcal{I}$ on which $u(\pi) > 0$. By complementarity, $\eta_1(\pi) = 0$ on $\mathcal{I}$, so the expression derived in Step 2 gives:

$$\int_{\pi_\mu}^{\pi} \eta_2(v)(v-\pi)\,dv + (1+(\lambda_a-\lambda_i))\int_{\pi_\mu}^{\pi} (F(v)-F(\pi_\mu))\,dv + (\lambda_a-\lambda_i)\int_{\pi_\mu}^{\pi} vf(v)\,dv = 0.$$

Differentiating twice with respect to π yields $\eta_2(\pi) = (1+2(\lambda_a-\lambda_i))f(\pi) + (\lambda_a-\lambda_i)\pi f'(\pi)$.

We now use the *transversality conditions* at $\pi = 1$: $\lambda_1(1) = \lambda_2(1) = 0$. Integrating the multiplier equations *from* π *to* 1:

$$\lambda_1(\pi) = \int_{\pi}^{1} (vf(v) - \eta_2(v)v)\,dv, \qquad \lambda_2(\pi) = \int_{\pi}^{1} (\eta_2(v) - (1+(\lambda_a-\lambda_i))f(v))\,dv.$$

Hence $\eta_1(\pi) = -\lambda_1(\pi) - \lambda_2(\pi)\pi = \int_\pi^1 \eta_2(v)(v-\pi)\,dv + (1+(\lambda_a-\lambda_i))\pi\int_\pi^1 f(v)\,dv - \int_\pi^1 vf(v)\,dv \geq 0$ for $\pi < 1$, consistently with the complementarity condition.

Case A: $\eta_2(\pi) = 0$ on a sub-interval $[\pi_0, \pi_1] \subset \mathcal{I}$. Then the expression $\eta_2(\pi) = (1 + 2(\lambda_a - \lambda_i))f(\pi) + (\lambda_a - \lambda_i)\pi f'(\pi) = 0$ is an ODE whose solution is $f(\pi) = C\pi^{-(2+1/(\lambda_a-\lambda_i))}$ on $[\pi_0, \pi_1]$. Since $\lambda_a > \lambda_i$ (established in Step 2 above), the exponent satisfies $-(2 +$

$1/(\lambda_a - \lambda_i)) < -3$. A log-concave density f satisfies $(\log f)'' \leq 0$, i.e. $f''f - (f')^2 \leq 0$. For $f(\pi) = C\pi^\gamma$ with $\gamma < -3$, we have $(\log f)'' = -\gamma/\pi^2 > 0$, which violates log-concavity. This yields a contradiction whenever f is log-concave (a condition that holds for the uniform, normal, exponential, and most standard distributions).

Case B: $\eta_2(\pi) > 0$ on $\mathcal{I}$. By complementarity, $(1 - \rho(\pi))\pi - (I - \sigma(\pi)) = 0$ on $\mathcal{I}$. Differentiating with respect to π: $-\rho'(\pi)\pi + (1 - \rho(\pi)) + \sigma'(\pi) = 0$. Substituting $\sigma'(\pi) = \pi u(\pi) = \pi\rho'(\pi)$ from $(IC_a^{(1)})$ gives $1 - \rho(\pi) = 0$, so $\rho(\pi) = 1$ and $\sigma(\pi) = I$, implying $u(\pi) = \rho'(\pi) = 0$—a contradiction with $u > 0$.

In both cases we reach a contradiction. We conclude that $u(\pi) = 0$ for all $\pi \geq \pi_\mu$, which establishes that $\rho' = \sigma' = 0$, i.e. the optimal allocation is uniform.

7.4 Proof of Proposition 3 (deal-by-deal characterization).

Step 1: The investor's rationality constraint binds

Since the agency is profit-maximizing, the investor's rationality constraint is binding. Otherwise, since the investor market is fully competitive, the agency could contract with a cheaper investor, thereby making more profit:

$$1 - \rho^* > \frac{I - \sigma^*}{\pi_\mu^*} \Rightarrow \exists \rho^\# \in \Big(\rho^*, 1 - \frac{I - \sigma^*}{\pi_\mu^*}\Big),$$
$$\int_{\pi_\mu^*}^1 (\rho^*\pi - \sigma^*) f(\pi)\, d\pi < \int_{\pi_\mu^*}^1 (\rho^\#\pi - \sigma^*) f(\pi)\, d\pi$$

and $(\sigma^*, \rho^\#)$ satisfies (BC_a), (BC_i), and $(IR_{i,p})$. Contradiction. Thus:

$$\rho^* = 1 - \frac{I - \sigma^*}{\pi_\mu^*}$$

Step 2: Reducing the agency's optimization problem

The agency solves:

$$\begin{aligned}
&\max_{\pi_\mu, \sigma} \int_{\pi_\mu}^1 \Big(\Big(1 - \frac{I - \sigma}{\pi_\mu}\Big)\pi - \sigma\Big) f(\pi)\, d\pi \\
= &\max_{\pi_\mu, \sigma} \int_{\pi_\mu}^1 \Big(\pi - \frac{I\pi}{\pi_\mu} + \frac{\sigma\pi}{\pi_\mu} - \sigma\Big) f(\pi)\, d\pi
\end{aligned}$$

under (BC_a) and (BC_i).

The Lagrangian is:

$$L = \int_{\pi_\mu}^1 \Big(\pi - \frac{I\pi}{\pi_\mu} + \frac{\sigma\pi}{\pi_\mu} - \sigma\Big) f(\pi)\, d\pi + \lambda_1 \Big(B_a - \sigma \int_{\pi_\mu}^1 f(\pi)\, d\pi\Big) + \lambda_2 \Big(B_i - (I - \sigma) \int_{\pi_\mu}^1 f(\pi)\, d\pi\Big)$$

$$\frac{\partial L}{\partial \sigma} = 0 \Leftrightarrow \int_{\pi_\mu}^{1} \Big(\frac{\pi}{\pi_\mu} - 1\Big) f(\pi)\, d\pi - \lambda_1 \int_{\pi_\mu}^{1} f(\pi)\, d\pi + \lambda_2 \int_{\pi_\mu}^{1} f(\pi)\, d\pi = 0$$

$$\frac{\partial L}{\partial \sigma} = 0 \Leftrightarrow \lambda_1 - \lambda_2 = \frac{\int_{\pi_\mu}^{1} (\pi - \pi_\mu) f(\pi)\, d\pi}{\pi_\mu \int_{\pi_\mu}^{1} f(\pi)\, d\pi} > 0$$

Thus $\lambda_1 > 0$ and $B_a = \sigma^* \int_{\pi_\mu^*}^{1} f(\pi)\, d\pi$.

$$\begin{aligned}
\frac{\partial L}{\partial \pi_\mu} = 0 &\Leftrightarrow -(\pi_\mu - I) f(\pi_\mu) + \int_{\pi_\mu}^{1} \pi(\sigma - I) \frac{\partial}{\partial \pi_\mu}\Big(\frac{1}{\pi_\mu}\Big) f(\pi)\, d\pi + \lambda_1 \sigma f(\pi_\mu) + \lambda_2 (I - \sigma) f(\pi_\mu) = 0 \\
&\Leftrightarrow -(\pi_\mu - I) f(\pi_\mu) + (\lambda_1 - \lambda_2)\sigma f(\pi_\mu) + \lambda_2 I f(\pi_\mu) + \frac{I - \sigma}{\pi_\mu^2} \int_{\pi_\mu}^{1} \pi f(\pi)\, d\pi = 0 \\
&\Leftrightarrow (\pi_\mu - I) f(\pi_\mu) = \\
&\Big(\frac{\int_{\pi_\mu}^{1} (\pi - \pi_\mu) f(\pi)\, d\pi}{\pi_\mu \int_{\pi_\mu}^{1} f(\pi)\, d\pi}\Big)\Big(\frac{B_a}{\int_{\pi_\mu}^{1} f(\pi)\, d\pi}\Big) f(\pi_\mu) + \lambda_2 I f(\pi_\mu) + \frac{I - \frac{B_a}{\int_{\pi_\mu}^{1} f(\pi)\, d\pi}}{\pi_\mu^2} \int_{\pi_\mu}^{1} \pi f(\pi)\, d\pi
\end{aligned}$$

Thus $\pi_\mu^* > I$.

Case 1 (cash-rich, under (A3)): $B_i > (I - \sigma^*) \int_{\pi_\mu^*}^{1} f(\pi)\, d\pi$, i.e. $\pi_\mu^* > (1 - F)^{-1}(\frac{B_a + B_i}{I})$.

Under assumption $(A3)$, the total budget is sufficient to finance all positive-NPV projects, so the investor's budget constraint (BC_i) is slack: $\lambda_2 = 0$. This is the relevant case.

The first-order condition on π_μ becomes:

$$(\pi_\mu - I) f(\pi_\mu) - \Big(\frac{\int_{\pi_\mu}^{1} (\pi - \pi_\mu) f(\pi)\, d\pi}{\pi_\mu \int_{\pi_\mu}^{1} f(\pi)\, d\pi}\Big)\Big(\frac{B_a}{\int_{\pi_\mu}^{1} f(\pi)\, d\pi}\Big) f(\pi_\mu) - \frac{I - \frac{B_a}{\int_{\pi_\mu}^{1} f(\pi)\, d\pi}}{\pi_\mu^2} \int_{\pi_\mu}^{1} \pi f(\pi)\, d\pi = 0$$

Since the agency's reduced objective $V(\pi_\mu)$ is continuous on the compact set $[I, \bar{p}]$ (where $\bar{p}$ is the largest feasible threshold satisfying $\sigma^* = B_a / \bar{F}(\pi_\mu) \leq I$), it attains a maximum. At any interior maximum π_μ^*, the first-order condition holds. The right-hand side is strictly positive, so $\pi_\mu^* > I$. The optimal allocation is then $\sigma^* = B_a / \bar{F}(\pi_\mu^*)$ and $\rho^* = 1 - (I - \sigma^*)/\pi_\mu^*$.

Case 2 (cash-poor boundary): $\lambda_2 = 0$ and $B_i = (I - \sigma^*) \int_{\pi_\mu^*}^{1} f(\pi)\, d\pi$.

This is the boundary of the cash-poor regime. Here the unconstrained optimum coincides with the budget-constrained threshold: $\pi_\mu^* = (1 - F)^{-1}(\frac{B_a + B_i}{I})$.

Case 3 (cash-poor): $\lambda_2 > 0$ and $B_i = (I - \sigma^*) \int_{\pi_\mu^*}^{1} f(\pi)\, d\pi$.

Both budget constraints bind. Thus:

$$\pi_\mu^* = (1-F)^{-1}\Big(\frac{B_a+B_i}{I}\Big), \quad \sigma^* = \frac{B_a}{B_a+B_i}I$$

The condition $\lambda_2 > 0$ imposes:

$$\begin{aligned}&(\pi_\mu^* - I)\, f(\pi_\mu^*)\\ &- \left(\frac{\int_{\pi_\mu^*}^1 (\pi - \pi_\mu^*)\, f(\pi)\, d\pi}{\pi_\mu^* \int_{\pi_\mu^*}^1 f(\pi)\, d\pi}\right)\left(\frac{B_a}{\int_{\pi_\mu^*}^1 f(\pi)\, d\pi}\right) f(\pi_\mu^*)\\ &- \left(\frac{I - \frac{B_a}{\int_{\pi_\mu^*}^1 f(\pi)\, d\pi}}{(\pi_\mu^*)^2}\right)\int_{\pi_\mu^*}^1 \pi\, f(\pi)\, d\pi\\ &= \lambda_2\, I\, f(\pi_\mu^*) \;>\; 0.\end{aligned}$$

The left-hand side is strictly positive, meaning the budget-constrained threshold $(1-F)^{-1}(\frac{B_a+B_i}{I})$ is *above* the unconstrained optimum from Case 1. The binding budget forces the agency to finance even fewer projects than the informational friction alone would dictate.

7.5 Uniqueness of the deal-by-deal threshold under f uniform.

For $f \equiv 1$ on $[0,1]$: $1-F(p) = 1-p$ and $\int_p^1 \pi\, d\pi = (1-p^2)/2$. Substituting $\sigma^* = B_a/(1-p)$ and $\rho^* = 1-(I-\sigma^*)/p$ into the agency's payoff $\int_p^1 (\rho^*\pi - \sigma^*) f(\pi)\, d\pi$, one obtains:

$$V(p) = \frac{(1-p)}{2p}\big[p^2 + p(1-I) - I + B_a\big].$$

Differentiating:

$$V'(p) = \frac{-2p^3 + Ip^2 + I - B_a}{2p^2}, \qquad V''(p) = -\frac{p^3 + I - B_a}{p^3}.$$

Since $p > I > 0$ and $B_a < I$ (from A2), $p^3 + I - B_a > 0$, so $V''(p) < 0$ for all $p > I$. The objective is strictly concave. Setting $V'(p) = 0$ gives the unique optimal threshold as the solution of $2p^3 - Ip^2 - (I - B_a) = 0$ in $(I, 1)$. □

7.6 Proof of Proposition 4 (blind-pool).

Step 1: The investor's rationality constraint binds in expectation

Since the agency is profit-maximizing, the investor's rationality constraint binds in expectation over $\{\pi > 0\}$. Otherwise, the agency could reduce the investor's share and increase its own profit. Formally, if (IR_i) were slack, there would exist $\hat{\rho}$ with a strictly higher agency share such that $(\sigma^*, \hat{\rho})$ still satisfies (BC_a), (BC_i), and (IR_i). Contradiction. Thus (IR_i) binds:

$$(1-\rho^*)\,\frac{\int_{\pi_\mu^*}^1 \pi f(\pi)\,d\pi}{\int_{\pi_\mu^*}^1 f(\pi)\,d\pi} = I - \sigma^*$$

Step 2: Reducing the agency's optimization problem

With the binding (IR_i), the agency's objective becomes:

$$\max_{\sigma,\pi_\mu} \int_{\pi_\mu}^1 \Big(\Big(1 - \frac{(I-\sigma)(1-F(\pi_\mu))}{\int_{\pi_\mu}^1 \pi f(\pi)\,d\pi}\Big)\pi - \sigma\Big) f(\pi)\,d\pi$$

under (BC_a) and (BC_i). Any allocation satisfying IC and the binding (IR_i) achieves the same welfare level, so the contract form is not uniquely determined.

Step 3: Finding the optimal investment strategy

The first-order condition with respect to π_μ^* gives:

$$\begin{aligned}
&\frac{\partial}{\partial \pi_\mu^*}\Big(\int_{\pi_\mu^*}^1 \nu^*(\pi) f(\pi)\,d\pi\Big) = 0 \\
\Leftrightarrow\quad &\frac{\partial}{\partial \pi_\mu^*}\Big(\int_{\pi_\mu^*}^1 \Big(\Big(1 - \frac{(I-\sigma^*)(1-F(\pi_\mu^*))}{\int_{\pi_\mu^*}^1 \pi f(\pi)\,d\pi}\Big)\pi - \sigma^*\Big) f(\pi)\,d\pi\Big) = 0
\end{aligned}$$

Expanding this derivative using the Leibniz rule and the quotient rule for the conditional expectation term:

$$\begin{aligned}
\frac{\partial \mathbb{E}[\pi \mid \pi \geq \pi_\mu^*]}{\partial \pi_\mu^*} &= \frac{\partial}{\partial \pi_\mu}\Big(\frac{\int_{\pi_\mu}^1 \pi f(\pi)\,d\pi}{1-F(\pi_\mu)}\Big) \\
&= \frac{-\pi_\mu f(\pi_\mu)(1-F(\pi_\mu)) + f(\pi_\mu)\int_{\pi_\mu}^1 \pi f(\pi)\,d\pi}{(1-F(\pi_\mu))^2} \\
&= \frac{f(\pi_\mu)\big(\int_{\pi_\mu}^1 \pi f(\pi)\,d\pi - \pi_\mu(1-F(\pi_\mu))\big)}{(1-F(\pi_\mu))^2}
\end{aligned}$$

After substitution and simplification, the first-order condition on π^*_μ reduces to:

$$
\begin{aligned}
&-\Big(\frac{(\pi^*_\mu - \sigma^*)\mathbb{E}[\pi \mid \pi \geq \pi^*_\mu] - \pi^*_\mu(I - \sigma^*)}{\mathbb{E}[\pi \mid \pi \geq \pi^*_\mu]}\Big) f(\pi^*_\mu) \\
&\qquad + \int_{\pi^*_\mu}^{1} \frac{\pi(I - \sigma^*)\frac{f(\pi^*_\mu)\big(\int_{\pi^*_\mu}^{1} \pi f(\pi)\, d\pi - \pi^*_\mu(1 - F(\pi^*_\mu))\big)}{(1 - F(\pi^*_\mu))^2}}{[\mathbb{E}(\pi \mid \pi \geq \pi^*_\mu)]^2} f(\pi)\, d\pi = 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & \Big((\sigma^* - \pi^*_\mu) + \frac{\pi^*_\mu(I - \sigma^*)\int_{\pi^*_\mu}^{1} f(\pi)\, d\pi}{\int_{\pi^*_\mu}^{1} \pi f(\pi)\, d\pi}\Big) f(\pi^*_\mu) \\
& \quad + \frac{(I - \sigma^*)\int_{\pi^*_\mu}^{1} (\pi - \pi^*_\mu) f(\pi)\, d\pi}{\int_{\pi^*_\mu}^{1} \pi f(\pi)\, d\pi} f(\pi^*_\mu) = 0 \\
\Leftrightarrow \quad & (-\pi^*_\mu + I) f(\pi^*_\mu) = 0
\end{aligned}
$$

Since $f(\pi^*_\mu) > 0$, we obtain $\pi^*_\mu = I$: all positive-NPV projects are financed.

The first-order condition with respect to σ^* is:

$$
\begin{aligned}
& \frac{\partial}{\partial \sigma^*}\Big(\int_{\pi^*_\mu}^{1} \nu^*(\pi) f(\pi)\, d\pi\Big) = 0 \\
\Leftrightarrow & \int_{\pi^*_\mu}^{1} \Big(\frac{\pi}{\mathbb{E}[\pi \mid \pi \geq \pi^*_\mu]} - 1\Big) f(\pi)\, d\pi = 0 \\
\Leftrightarrow & \, \mathbb{E}[\pi \mid \pi \geq \pi^*_\mu] = \frac{\int_{\pi^*_\mu}^{1} \pi f(\pi)\, d\pi}{1 - F(\pi^*_\mu)}
\end{aligned}
$$

which is true by definition. Hence σ^* is not pinned down by the unconstrained first-order conditions alone; it is determined by the budget constraints.

The Lagrangian is:

$$
\begin{aligned}
L(\sigma, \pi_\mu, \lambda_1, \lambda_2) = & -\int_{\pi_\mu}^{1} \Big(\Big(1 - \frac{(I - \sigma)(1 - F(\pi_\mu))}{\int_{\pi_\mu}^{1} \pi f(\pi)\, d\pi}\Big)\pi - \sigma\Big) f(\pi)\, d\pi \\
& + \lambda_1\Big(\sigma \int_{\pi_\mu}^{1} f(\pi)\, d\pi - B_a\Big) + \lambda_2\Big((I - \sigma)\int_{\pi_\mu}^{1} f(\pi)\, d\pi - B_i\Big)
\end{aligned}
$$

The stationarity conditions give:

$$
\begin{aligned}
& \frac{\partial L}{\partial \pi_\mu}(\sigma, \pi_\mu, \lambda_1, \lambda_2) = \big((\pi_\mu - I) - \lambda_1\sigma - \lambda_2(I - \sigma)\big) f(\pi_\mu) \\
& \frac{\partial L}{\partial \sigma}(\sigma, \pi_\mu, \lambda_1, \lambda_2) = (-\lambda_1 + \lambda_2)(1 - F(\pi_\mu))
\end{aligned}
$$

Setting $\frac{\partial L}{\partial \sigma} = 0$ gives $\lambda_1^* = \lambda_2^*$, and $\frac{\partial L}{\partial \pi_\mu} = 0$ gives $\pi_\mu^* - I - \lambda_1^* I = 0$.

Case 1 (cash-poor): $\lambda_1^* > 0$ and $\sigma^* \int_{\pi_\mu^*}^1 f(\pi)\, d\pi = B_a$

The agency exhausts all the available budget by saturating both budget constraints:

$$\lambda_1^* > 0 \Leftrightarrow \lambda_2^* > 0 \Leftrightarrow (I - \sigma^*) \int_{\pi_\mu^*}^1 f(\pi)\, d\pi = B_i$$

Thus:

$$\pi_\mu^* = (1-F)^{-1}\Big(\frac{B_a + B_i}{I}\Big), \quad \sigma^* = \frac{B_a}{\int_{(1-F)^{-1}(\frac{B_a+B_i}{I})}^1 f(\pi)\, d\pi} = \frac{B_a}{B_a + B_i} I$$

This case applies when:

$$\begin{aligned} \lambda_1^* = \frac{\pi_\mu^* - I}{I} > 0 &\Leftrightarrow (1-F)^{-1}\Big(\frac{B_a + B_i}{I}\Big) > I \\ &\Leftrightarrow B_a + B_i < I(1-F)(I) \end{aligned}$$

Case 2 (cash-rich): $\lambda_1^* = 0$ and $\sigma^* \int_{\pi_\mu^*}^1 f(\pi)\, d\pi < B_a$

Under $(A3)$, the available budget is sufficient to finance all positive-NPV projects:

$$\lambda_1^* = 0 \Leftrightarrow \lambda_2^* = 0 \Leftrightarrow (I - \sigma^*) \int_{\pi_\mu^*}^1 f(\pi)\, d\pi < B_i$$

and:

$$\pi_\mu^* - I - \lambda_1^* I = 0 \Leftrightarrow \pi_\mu^* = I$$

The agency's investment share satisfies:

$$\sigma^* \in \Big]\frac{B_a}{(1-F)(I)},\ I - \frac{B_i}{(1-F)(I)}\Big[\ \cap\ (0, I]$$

which requires:

$$B_a + B_i > I(1-F)(I)$$

This is precisely assumption $(A3)$. The uniform allocation (ρ^*, σ^*) with threshold $\pi_\mu^* = I$ is one optimal solution; any allocation satisfying IC and the binding (IR_i) achieves the same welfare.

Case 3 (knife-edge): $\lambda_1^* = \lambda_2^* = 0$, $\sigma^* \int_{\pi_\mu^*}^1 f(\pi)\, d\pi = B_a$ and $(I - \sigma^*) \int_{\pi_\mu^*}^1 f(\pi)\, d\pi < B_i$

Thus:

$$\pi_\mu^* = I, \quad \frac{B_a + B_i}{(1-F)(I)} = I$$

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